ANALYTICAL AND EXPERIMENTAL DETERMINATION OF THE CHARACTERISTICS OF A TRANSONIC AXIAL TURBINE

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THESIS

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OF THE CHARACTERISTICS OF A TRANSONIC
AXIAL TURBINE

by

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An analysis and test rig measurements of the performance of a transonic axial turbine are reported. The purpose was to confirm the accuracy of measurements made in a test rig which was designed to separate the losses occurring in the stator from the losses occurring in the rotor blade rows. The analysis was programmed for the Hewlett-Packard 21-MX computer. Reasonable agreement between predicted and measured characteristics was obtained using experimentally determined losses in the computer program. Lack of

agreement was noted using theoretical values. It was concluded that the rotor was not choked at the conditions in the tests, and that the test rig measurements were valid. A successful technique for smoothing the data obtained from the rig is also reported.

Analytical and Experimental Determination of
the Characteristics of a Transonic Axial Turbine

by

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ABSTRACT

An analysis and test rig measurements of the performance of a transonic axial turbine are reported. The purpose was to confirm the accuracy of measurements made in a test rig which was designed to separate the losses occurring in the stator from the losses occurring in the rotor blade rows. The analysis was programmed for the Hewlett-Packard 21-MX computer. Reasonable agreement between predicted and measured characteristics was obtained using experimentally determined losses in the computer program. Lack of agreement was noted using theoretical values. It was concluded that the rotor was not choked at the conditions in the tests, and that the test rig measurements were valid. A successful technique for smoothing the data obtained from the rig is also reported.

I. INTRODUCTION

The transonic turbine test rig installation at the Naval Postgraduate School Turbopropulsion Laboratory was designed to study the effects on turbine performance of varying axial and tip clearances, to study the effects of blading design on turbine performance, and to allow the separate determination of stator and rotor losses in an operating machine.

Until the work of Solms (Ref. 1) the separation of rotor and stator losses through test rig measurements had not been attained satisfactorily. Through improvements in hardware and instrumentation and improvements in the data reduction process the stator and rotor losses were determined separately and were reported in Ref. 1.

Anomalies remained, however. Specifically, the turbine configuration designated as Turbine C in Ref. 1 gave different results when compared in terms of "referred" quantities depending on whether the discharge was to atmospheric pressure or to a region of reduced pressure. (Turbine C had converging-diverging stator passages in an axial entry, single impulse stage. The turbine was designed to operate in the transonic range.) In addition, Ref. 1 reported considerable scatter in the loss coefficients.

In the work of Robbins (Ref. 2) it was shown that discharge pressure affected the measurements of flow rate into the stage and also affected the labyrinth leak rate.

Accordingly, Robbins determined accurately the flow rate into the stage and the leak rate through the labyrinth seal (See Fig. 1) for all operating conditions. The results then obtained for turbine C were reported fully in Ref. 2. The continued presence of scatter in the measured loss coefficients was reported and a smoothing technique to eliminate the scatter was suggested.

Before the Turbine Test Rig could be used to measure the effects of varying parameters:

1. The scatter in the loss coefficients had to be eliminated.
2. The overall accuracy of the performance results evaluated from the rig measurements had to be verified in some way.

The resolution of these problems was the goal of the present work and is the subject of this report. First, a satisfactory method was found for smoothing the loss coefficients. The method is described in Section III.

The approach taken to verify the performance of the rig was to first devise an analysis which predicted the performance of the turbine in terms of unknown loss coefficients, and then to show that the measured loss coefficients were consistent with the predicted behaviour.

A description of the Turbine Test Rig is given in Section II. The analysis of the behaviour of the turbine, involving a comparison of an analytical prediction with the results of a short test program, is described in

Section IV. Details of the analysis and the computer program are given in Appendix A.

II. TURBINE TEST RIG INSTALLATION

A. DESCRIPTION

The test installation consists of three major components; an Allis-Chalmers twelve stage axial flow compressor, an exhauster assembly, and the turbine test rig (TTR) itself.

The compressor is the source of driving air for the TTR and for the exhauster assembly. Fig. 2 shows the piping arrangement. Turbine air passes through the first settling tank into an eight-inch pipe containing a flow nozzle, into the second settling tank and into the turbine.

Fig. 3 shows the plenum, the floating stator assembly, the rotor, and the dynamometer (Ref. 1). Pressure ratios of 6:1 can be achieved when the system is hooded. The hood was needed to achieve high pressure ratios in the tests reported here. Fig. 4 shows the turbine blading of the stator and rotor. Ref. 3 contains detailed descriptions of the test rig hardware.

The floating stator assembly shown in Fig. 3 permits measurements of the axial force and the torque on the assembly. Axial and rotational movements are constrained by calibrated force transducers that are heat insensitive. These measurements, together with wall static pressure measurements, allow the determination of the average axial and tangential velocity components at the stator exit.

In this report one configuration designated Turbine C

was tested, the geometry of which is shown in Fig. 5. Table I describes the geometry quantitatively. The stator blade profile is shown in Fig. 6. The blades of the stator generate a converging-diverging nozzle shape. Pressure measurements were taken at the locations shown in Fig. 6. The pressures necessary to the analysis of the stator axial force were taken at the locations shown in Fig. 7.

B. TEST MEASUREMENTS AND ACCURACY

1. Mass Flow Rates

Appendix A of Ref. 2 gives a detailed description of the method used to determine both the turbine flow rate and the labyrinth seal leak rate.

2. Forces, Torques, Temperatures, and Pressure

Ref. 3 and Ref. 5 give calibration procedures for the TTR. Identical procedures were employed here. Table II of Ref. 1 gives the expected accuracies of the measurements.

C. TESTING AND DATA REDUCTION

The TTR data collection system is described in Ref. 4. Appendix D of Ref. 1 gives a detailed explanation of the turbine test procedures. Those procedures were followed here with the exception that a constant RPM was held and the pressure ratio varied over the desired range. The data reduction method developed in Ref. 1 and Ref. 2 was revised as described in Appendix B.

III. DATA SMOOTHING TECHNIQUE

The sensitivity of the loss coefficients to variations in measured quantities was shown in Ref. 2. It was also stated that some variation is unavoidable since measurements are taken over a period of more than a minute. It was also pointed out in Ref. 2 that the parameter most important to the calculation of the loss coefficients was P_1 (the average pressure at the stator exit). P_1 is an average pressure that can not be measured directly. It is derived as described in Ref. 1 from many other measurements. Significant scatter was observed in the variation of P_1 as speed was varied at fixed pressure ratio. However, it was found that the hub and tip pressures (P_h and P_t) measured just downstream of the stator varied smoothly over the same range. Since these two pressure were measured directly, and since they varied smoothly, it was assumed that the pressure behind the stator must vary smoothly also. Consequently, it was determined that a polynomial curve fit could be used to describe the variation of P_1 as speed was varied. The variation of P_1 was represented as a function of P_h, P_t, and the isentropic head coefficient (K_{is}) in the form:

$$\sigma = \frac{P_1 - P_h}{P_t - P_h} = A_0 + A_1 K_{is} + A_2 K_{is}^2 \cdots \qquad (1)$$

Where A_i is the polynomial coefficient.

Then, P_1/P_{to} (where P_{to} is total pressure upstream of the stator) was computed using the expression

$$\frac{P_1}{P_{to}} = \sigma\left(\frac{P_t - P_h}{P_{to}}\right) + \left(\frac{P_h}{P_{to}}\right)$$

Fig. 8 is an illustration of this procedure, which was added to the "bulk process" data reduction program. The data points in Fig. 8 were taken from Runs 6 and 7 in Ref. 2. The two Runs were for the same conditions but were made at different times. It can be seen that the trends are the same for both runs but the scatter is considerable. The scatter is the cause of the scatter in the calculated loss coefficients. The lines on Fig. 8 are the polynomial approximations according to Eq. (1) for the two runs. It can be seen that the polynomial approximation averages the data and maintains the original trend. The chosen smoothing function was a 1st degree polynomial.

Fig. 9 shows the stator loss coefficient prior to smoothing for the points in Run #7. Fig. 10 shows the same data after smoothing.

Similarly, Fig. 11 shows the rotor loss coefficients for Run #7 prior to smoothing and Fig. 12 shows the same data after smoothing.

It can be seen from these figures that the scatter has been removed. This is particularly true for the rotor loss coefficients. As pointed out above, it is believed that the observed scatter was due to the sensitivity of the loss coefficients to small changes in measured quantities during the data collection process. The smoothing technique removes the random variations recorded during

data collection and results in a much more realistic representation of the variation in the losses.

The smoothing technique was incorporated into the data reduction program as described in Appendix B.

IV. ANALYSIS OF TURBINE PERFORMANCE

A. APPROACH

In order to determine if the performance evaluated from the rig measurements was accurate an analysis to predict the behaviour of the test turbine was carried out and programmed in BASIC language.

A performance test was then conducted in a particular way in order to provide a comparison of the measured with the predicted behaviour.

B. ANALYTICAL PREDICTIONS

The flow through the turbine was analysed using a pseudo-1 Dimensional compressible approach. The analysis is described in detail in Appendix A, together with the computer program which was used to obtain predictions of the turbine performance. One of the inputs which the program requires is the rotor passage loss coefficient at zero incidence. Using the method given by Vavra in Ref. 6 it was determined that the rotor loss coefficient should have the value .2514. However, results of previous tests of the turbine indicated that the rotor loss coefficient was rarely as high as .2514 and could be as low as .1. Therefore, the performance of the turbine was analysed using rotor loss coefficients of .2514 and .1. The prediction program was run for both 15,000 and 18,000 RPM with an assumed rotor loss coefficient of .2514, and for 18,000 RPM with an assumed rotor loss coefficient of .1

The above parameters were chosen to obtain a prediction of the turbine performance in a range in which experimental data could be obtained. In particular, the analysis could be used to predict the pressure ratios at which choking would occur in the rotor as well as in the stator. The pressure ratio at which choking occurred in the rotor could then be established experimentally in a test conducted at fixed speed. An examination of the choking condition was considered to be a first test of the performance analysis.

The results of the analysis for the parmameters given above are shown in Fig. 13, Fig. 14, and Fig. 15. What is shown in the figures is the map of the range of values which unknown parameters in the analysis can have, that leads to a solution.

Figures 16-19 show predicted performance parameters for the case of 18,000 RPM and assumed rotor loss of .1. These results will be discussed in conjunction with the results of the turbine test run.

C. EXPERIMENTAL TURBINE TEST

In order to examine the occurrence of rotor choking the turbine was run at constant RPM and the pressure ratio across the stage was increased in increments by lowering the back pressure. The point at which the horsepower ceased to increase for an increase in pressure ratio was examined to determine the choking point. At the condition where the flow reaches a Mach number of unity at the exit of the rotor, the power produced by the turbine can not be changed

by altering the downstream pressure.

Tests were conducted in this manner at 15,000 and 18,000 RPM. The controlled parameters for the tests are given in Table II. The reduced data is given in Table III. The referred horsepower is shown plotted versus the pressure ratio for the 15,000 RPM run in Fig. 20 and for the 18,000 RPM run in Fig. 21.

The results are discussed in the next section.

D. COMPARISON OF ACTUAL AND PREDICTED PERFORMANCE

Figures 13, 14, and 15 are maps of possible solutions for the flow through the turbine, when the parameters which are unknown are allowed to vary. It is noted first that the predicted range of solutions extends to pressure ratios (P_2/P_{to}) below the predicted choking line. The explanation for this apparent anomaly is that the program calculates all possible solutions, and for any point below the line a particular combination of losses and blockage factors existed which would allow a solution at that point without choking. It should be pointed out that any point on the plot can be brought to the choking line by reducing the stator loss by a very small amount. This is the procedure that was followed to get the range of values at choking shown in Figures 16-19.

Fig. 13 is the map produced by the program for RPM equal 15,000 and for an assumed rotor passage loss coefficient equal to .2514. As can be seen from the figure the pressure ratio, (P_2/P_{to}), at choking was about .28, corresponding to a stage pressure ratio (P_{to}/P_2) of 3.57. Fig. 14 is the map for RPM=18,000. Note that the predicted stage pressure ratio for choking is again about 3.57.

Fig. 20 and Fig. 21 show the variation of the referred horsepower vs. the pressure ratio which were measured in turbine tests at 15,000 and 18,000 RPM respectively. It can be seen that the referred horsepower did not become independent of the back pressure at the pressure ratios

achieved in these tests. It was concluded that the stage did not choke at the predicted pressure ratio of 3.57. It was noted however that at the highest pressure ratios obtained at 18,000 RPM, the slope of the horsepower curve was becoming smaller.

The velocity diagrams in Figures 23-31 are also consistent with the argument that the rotor did not choke during the turbine tests. The velocity from the rotor, V_2, behaves smoothly and in a predictable manner throughout the pressure range at both test speeds. This might not be expected if, following choking, shock waves appeared downstream of the rotor exit plane.

Note the values of rotor loss coefficient given for the test results in Table III. With the exception of point #10, which was considered to be in error, they were all smaller than the value (.2514) assumed in the first performance calculations. As explained in Appendix A, the value of the rotor passage loss coefficient used in the program is the smallest value that can be calculated for the overall rotor loss coefficient. Therefore, if the experimental results were accurate, the computer program could not predict the correct performance since it could never calculate a rotor loss less than the input value of the passage loss, which was .2514. It was as a consequence of this observation that the performance was re-calculated using a rotor passage loss coefficient equal to .1.

Figures 16-19 show the choking behaviour of the

turbine predicted using the computer program for an assumed rotor passage loss of .1. In these figures, the data for test point #6 is also shown. In this case it can be seen that the predicted pressure ratio (P_2/P_{to}) at choking is about .245, corresponding to a stage pressure ratio (P_{to}/P_2) of about 4.08. The latter is slightly higher than the maximum pressure ratio that was attained in the turbine tests. (At 18,000 RPM the highest attainable pressure ratio was 3.97, due to the characteristics of the dynamometer.)

Fig. 17 is the range of efficiencies predicted for the choked condition. It is noted that the efficiency measured at test point #6 was reasonably close to the predicted maximum efficiency.

Fig. 18 shows the range of stator losses for which solutions existed, in comparison with the value of stator loss measured at test point #6. It can be seen that the measured value intersects the predicted range of possible solutions. Fig. 19 shows the predicted range of the rotor coefficient. Again, it can be seen that the measured value at test point #6 overlaps the predicted range of possible solutions.

Fig. 16 shows the predicted range of horsepower compared with the horsepower measured at test point #6. The measured horsepower was slightly greater than the maximum value which was predicted.

E. DISCUSSION

The results shown in Figures 16-19 illustrate the uncertainty in the prediction of the performance of the turbine when only the rotor passage loss and blockage factor are known. It is recalled that the analysis satisfies only continuity through the stage, and values of an additional loss coefficient and an additional blockage factor must be established to obtain a unique solution. It is the aerodynamic shaping of the surfaces which determines these factors. Figures 16-19 show the possible range of performance that results simply from the areas of the passages and the blade angles.

The irregular shapes of the bounds on the possible solutions in Figures 16-19 are interesting. There is no obvious explanation for the reduced range of solutions near $k_{b1} = .81$.

The results obtained in the present work suggest that the performance of the turbine as measured in the turbine test rig is reliable. It had been thought previously that the rotor loss measurements were too low. Here, an analysis was carried out to predict the performance of the turbine, and the predicted results have shown very good agreement with the performance measurements in the test rig.

Whether or not the rotor was choking had also been a question in the past. In the present work both the analysis and experimental tests have shown that the rotor was not

choked in any test to date. The computer program with an assumed rotor loss of .1 has shown that the test rig should not be choked at any pressure ratio below about 4.08. Test data has shown that the rotor was not choked at a pressure ratio just slightly below 4.0.

It is also of interest to note that for a rotor passage loss coefficient of .2514 the predicted pressure ratio was shown to be independent of RPM. At both 15,000 and 18,000 RPM the choking pressure ratio (P_{to}/P_2) was calculated to be about 3.57. More results are needed to confirm that the choking pressure ratio is indeed independent of speed.

V. CONCLUSIONS AND RECOMMENDATIONS

1. The rotor of Turbine C was not choked at any pressure ratio tested so far. This conclusion is based on the prediction of the computer program and also on the turbine test results.

2. The loss coefficients measured in the TTR and smoothed as described in this report can be accepted as being truly representative of the losses in the separate blade rows of the turbine.

3. In particular, the results from the computer program suggest that the magnitude of the measured rotor loss coefficients is probably correct. The method used by Vavra in Ref. 6 predicts much larger values of loss coefficients for the present rotor geometry than those which were measured. When the measured value of the rotor loss coefficient was entered into the computer program, there was good agreement between all the calculated and the measured turbine performance parameters. When the higher (calculated) value of the rotor loss was entered, there was a pronounced disagreement between the calculated and the measured performance.

4. The computer program should be used (and progressively developed) in conjunction with all tests carried out in the turbine test rig. Most importantly, a test

should be conducted to determine the rotor choking condition experimentally. This will require the purchase of a water-brake dynamometer to extend the power-speed range of the test rig.

5. Since the accuracy of the test rig measurements is no longer in doubt, experiments to determine the effect of parameter changes (e.g. axial and tip clearances) on turbine performance can go ahead.

TABLE I

TURBINE GEOMETRY

	TURBINE	STATOR		ROTOR	
	C	1		1	
	DESCRIPTION	A* (in^2)	NUMBER OF BLADES	R_M (IN)	h (in.)
STATOR 1	CONVERGING DIVERGING NOZZLES	2.9058	31	4.184	0.5775
ROTOR 1	CIRCULAR ARC	7.119	60	$R_{m1}=4.193$ $R_{m2}=4.250$	$h_1=0.732$ $h_2=0.8475$

TABLE II

PARAMETERS FOR TURBINE TEST

POINT #	RPM	P_{to}/P_2
1	15,000	2.01
2	15,000	2.42
3	15,000	2.93
4	15,000	3.49
5	15,000	3.68
6	18,000	3.94
7	18,000	3.47
8	18,000	2.98
9	18,000	2.52
10	18,000	2.02

TABLE III TURBINE TEST RESULTS

POINT #1

```
PRESSURE RATIO         = 2.011859838
REF FLOW RATE          = 1.020000000
REF RPM                = 14262.15617
REF ROTOR MOMENT       = 102.5192926
REF HP                 = 23.19933152
EFF. T-S               = 0.712728663

STATOR LOSS THEOR.  = 0.107100795
STATOR LOSS COEFF.  = 0.220295023
ROTOR LOSS COEFF.   = 0.198187353
ROTOR LOSS THEOR.   = 0.386101836
----------------------------------------
ISEN. HEAD COEFF.      = 4.036270755
TH. DEG. OF REACTION = 0.025306644
ACT. DEG. OF REACTION=-0.084678182

VELOCITY TRIANGLE DATA

V1 =  973.73327    V2 =  204.23594
VA1=  226.65467    VA2=  155.42281
VU1=  946.98688    VU2=  132.49932

ALPHA 1=   76.53987   ALPHA 2=   40.44789
BETA 1 =   60.44410   BETA 2 =  -69.84574
```

POINT #2

```
PRESSURE RATIO         = 2.420527383
REF FLOW RATE          = 1.020000000
REF RPM                = 14309.0588
REF ROTOR MOMENT       = 132.334797
REF HP                 = 30.04483385
EFF. T-S               = 0.748970950

STATOR LOSS THEOR.  = 0.107100795
STATOR LOSS COEFF.  = 0.183632754
ROTOR LOSS COEFF.   = 0.183537254
ROTOR LOSS THEOR.   = 0.455681440
----------------------------------------
ISEN. HEAD COEFF.      = 4.941765312
TH. DEG. OF REACTION =-8.76562E-03
ACT. DEG. OF REACTION=-0.272796901

VELOCITY TRIANGLE DATA

V1 = 1126.06784    V2 =  184.36934
VA1=  265.46949    VA2=  178.98521
VU1= 1094.32844    VU2=   44.23066

ALPHA 1=   76.36422   ALPHA 2=   13.88078
BETA 1 =   64.02313   BETA 2 =  -70.79960
```

TABLE III TURBINE TEST RESULTS (CONT.)

POINT #3

```
PRESSURE RATIO       = 2.93257874
REF FLOW RATE        = 1.020000000
REF RPM              = 14297.06424
REF ROTOR MOMENT     = 161.7256587
REF HP               = 36.6868547
EFF. T-S             = 0.769522864

STATOR LOSS THEOR. = 0.107100795
STATOR LOSS COEFF. = 0.172065787
ROTOR LOSS COEFF.  = 0.093520223
ROTOR LOSS THEOR.  = 0.43179246
----------------------------------------------------------------
ISEN. HEAD COEFF.     = 5.882942922
TH. DEG. OF REACTION =-0.031239189
ACT. DEG. OF REACTION=-0.241795592

VELOCITY TRIANGLE DATA

V1 = 1248.51552    V2 =  218.93763
VA1=  314.71895    VA2=  206.88375
VU1= 1208.19824    VU2=  -71.64356

ALPHA 1=   75.39969   ALPHA 2=  -19.10090
BETA 1 =   64.49981   BETA 2 =  -71.78607
```

POINT #4

```
PRESSURE RATIO       = 3.490951547
REF FLOW RATE        = 1.020000000
REF RPM              = 14290.27554
REF ROTOR MOMENT     = 181.5713712
REF HP               = 41.16922187
EFF. T-S             = 0.758539626

STATOR LOSS THEOR. = 0.107100795
STATOR LOSS COEFF. = 0.158068259
ROTOR LOSS COEFF.  = 0.134491185
ROTOR LOSS THEOR.  = 0.427987152
----------------------------------------------------------------
ISEN. HEAD COEFF.     = 6.703669585
TH. DEG. OF REACTION =-0.014684530
ACT. DEG. OF REACTION=-0.213884738

VELOCITY TRIANGLE DATA

V1  = 1333.15077   V2 =  282.31378
VA1=  355.00633    VA2=  238.26055
VU1= 1285.01420    VU2= -151.43638

ALPHA 1=   74.55631   ALPHA 2=  -32.43969
BETA 1 =   64.26912   BETA 2 =  -71.41323
```

TABLE III TURBINE TEST RESULTS (CONT.)

POINT #5

```
PRESSURE RATIO       = 3.677359655
REF FLOW RATE        = 1.020000000
REF RPM              = 14292.77432
REF ROTOR MOMENT     = 186.8682436
REF HP               = 42.3776356
EFF. T-S             = 0.752803647

STATOR LOSS THEOR. = 0.107100795
STATOR LOSS COEFF. = 0.140979499
ROTOR LOSS COEFF.  = 0.192867968
ROTOR LOSS THEOR.  = 0.435423887
--------------------------------------------------------------
ISEN. HEAD COEFF.     = 6.950584955
TH. DEG. OF REACTION = 3.90802E-03
ACT. DEG. OF REACTION=-0.223696759

VELOCITY TRIANGLE DATA

V1  = 1362.18537   V2 =  301.42998
VA1=  361.10549   VA2=  250.00963
VU1= 1313.45034   VU2= -168.39007

ALPHA 1=   74.62753   ALPHA 2=  -33.96162
BETA 1 =   64.69083   BETA 2 =  -71.02123
```

POINT #6

```
PRESSURE RATIO       = 3.938522427
REF FLOW RATE        = 1.020000000
REF RPM              = 17091.82469
REF ROTOR MOMENT     = 169.2232867
REF HP               = 45.89160403
EFF. T-S             = 0.784488861

STATOR LOSS THEOR. = 0.107100795
STATOR LOSS COEFF. = 0.141347589
ROTOR LOSS COEFF.  = 0.187722740
ROTOR LOSS THEOR.  = 0.421689292
--------------------------------------------------------------
ISEN. HEAD COEFF.     = 5.050902877
TH. DEG. OF REACTION = 0.104266156
ACT. DEG. OF REACTION= 0.175212937

VELOCITY TRIANGLE DATA

V1  = 1318.03669   V2 =  269.74423
VA1=  334.28819   VA2=  260.54759
VU1= 1274.94005   VU2=  -69.83484

ALPHA 1=   75.30782   ALPHA 2=  -15.0043[illegible]
BETA 1 =   61.53822   BETA 2 =  -70.56818
```

TABLE III TURBINE TEST RESULTS (CONT.)

POINT #7

```
PRESSURE RATIO       = 3.467558140
REF FLOW RATE        = 1.020000000
REF RPM              = 17039.50815
REF ROTOR MOMENT     = 155.9660642
REF HP               = 42.16691707
EFF. T-S             = 0.784673443

STATOR LOSS THEOR. = 0.107100795
STATOR LOSS COEFF. = 0.165054929
ROTOR LOSS COEFF.  = 0.128352524
ROTOR LOSS THEOR.  = 0.418863835
--------------------------------------------------------------
ISEN. HEAD COEFF.      = 4.668401576
TH. DEG. OF REACTION = 0.051715826
ACT. DEG. OF REACTION= 0.080815056

VELOCITY TRIANGLE DATA

V1 = 1280.67141    V2 =  234.80211
VA1=  326.98646    VA2=  234.79915
VU1= 1238.22418    VU2=   -1.17934

ALPHA 1=   75.20720   ALPHA 2=   -0.28778
BETA 1 =   60.69243   BETA 2 =  -70.61495
```

POINT #8

```
PRESSURE RATIO       = 2.974737893
REF FLOW RATE        = 1.020000000
REF RPM              = 17058.70933
REF ROTOR MOMENT     = 137.689044
REF HP               = 37.26749796
EFF. T-S             = 0.774777057

STATOR LOSS THEOR. = 0.107100795
STATOR LOSS COEFF. = 0.189140155
ROTOR LOSS COEFF.  = 0.083159985
ROTOR LOSS THEOR.  = 0.414274730
--------------------------------------------------------------
ISEN. HEAD COEFF.      = 4.169275243
TH. DEG. OF REACTION = 0.038020949
ACT. DEG. OF REACTION= 0.096859972

VELOCITY TRIANGLE DATA

V1 = 1200.60325    V2 =  219.93553
VA1=  298.71298    VA2=  208.69610
VU1= 1162.84940    VU2=   69.40873

ALPHA 1=   75.59334   ALPHA 2=   18.39624
BETA 1 =   59.51871   BETA 2 =  -70.71206
```

TABLE III TURBINE TEST RESULTS (CONT.)

POINT #9

```
PRESSURE RATIO       = 2.519408034
REF FLOW RATE        = 1.020000000
REF RPM              = 17170.86109
REF ROTOR MOMENT     = 113.7867884
REF HP               = 31.00049275
EFF. T-S             = 0.743362897

STATOR LOSS THEOR. = 0.107100795
STATOR LOSS COEFF. = 0.189633321
ROTOR LOSS COEFF.  = 0.191292857
ROTOR LOSS THEOR.  = 0.413068844
-----------------------------------------------------------------
ISEN. HEAD COEFF.     = 3.567656122
TH. DEG. OF REACTION = 0.058183678
ACT. DEG. OF REACTION= 0.108307260

VELOCITY TRIANGLE DATA

V1 = 1103.46913   V2 =  249.75075
VA1=  259.39382   VA2=  183.84024
VU1= 1072.54789   VU2=  169.05088

ALPHA 1=   76.40418   ALPHA 2=   42.60019
BETA 1 =   57.94743   BETA 2 =  -69.80025
```

POINT #10

```
PRESSURE RATIO       = 2.018415708
REF FLOW RATE        = 1.020000000
REF RPM              = 17158.62871
REF ROTOR MOMENT     = 82.24688045
REF HP               = 22.39168077
EFF. T-S             = 0.688180960

STATOR LOSS THEOR. = 0.107100795
STATOR LOSS COEFF. = 0.114113509
ROTOR LOSS COEFF.  = 0.489230808
ROTOR LOSS THEOR.  = 0.351684730
-----------------------------------------------------------------
ISEN. HEAD COEFF.     = 2.787522027
TH. DEG. OF REACTION = 0.114732223
ACT. DEG. OF REACTION=-1.15697E-03

VELOCITY TRIANGLE DATA

V1 =  987.09080   V2 =  345.21705
VA1=  210.24041   VA2=  153.85815
VU1=  964.44140   VU2=  309.03475

ALPHA 1=   77.70237   ALPHA 2=   63.53281
BETA 1 =   55.61835   BETA 2 =  -66.77659
```

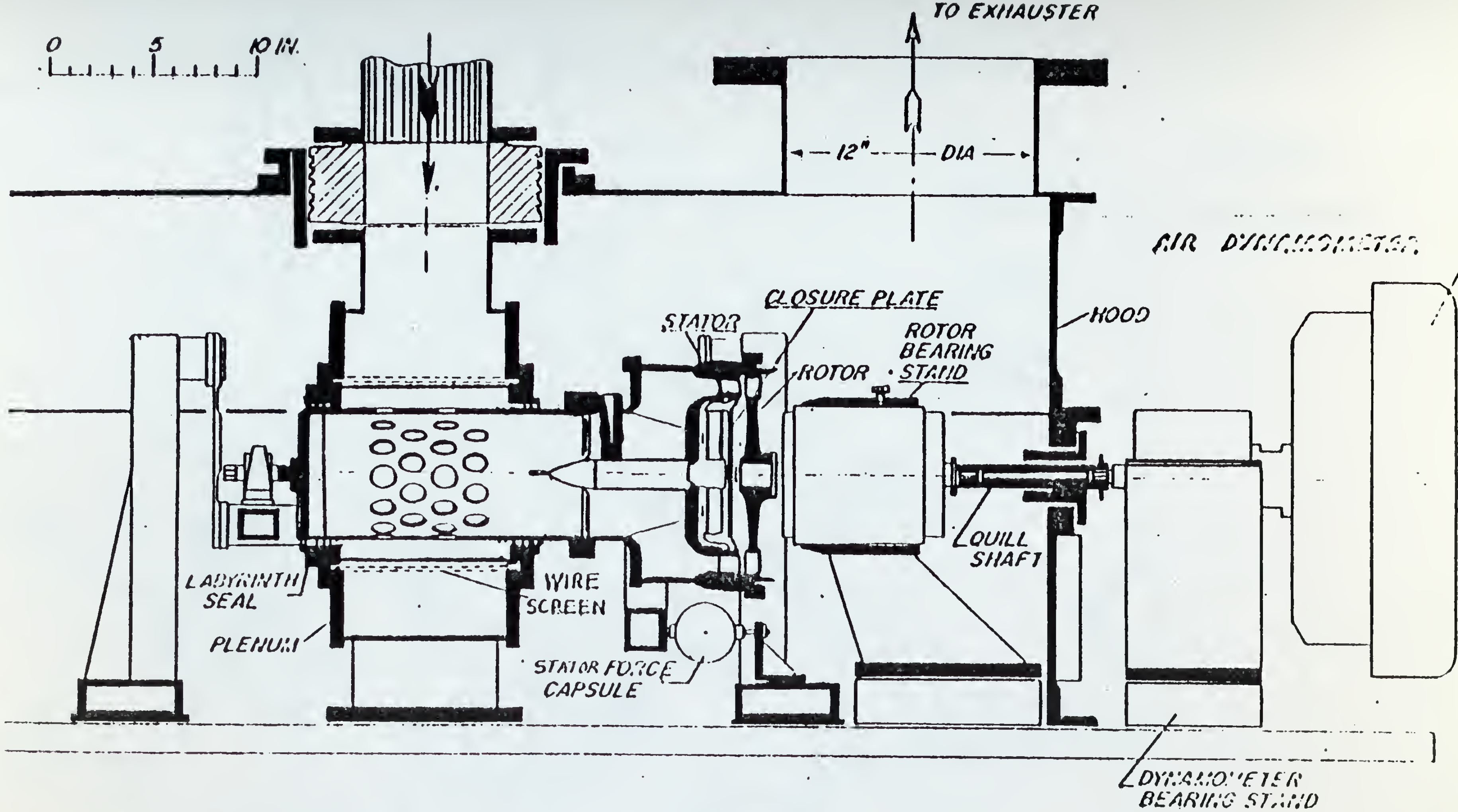


FIGURE 1 TRANSONIC TURBINE TEST RIG

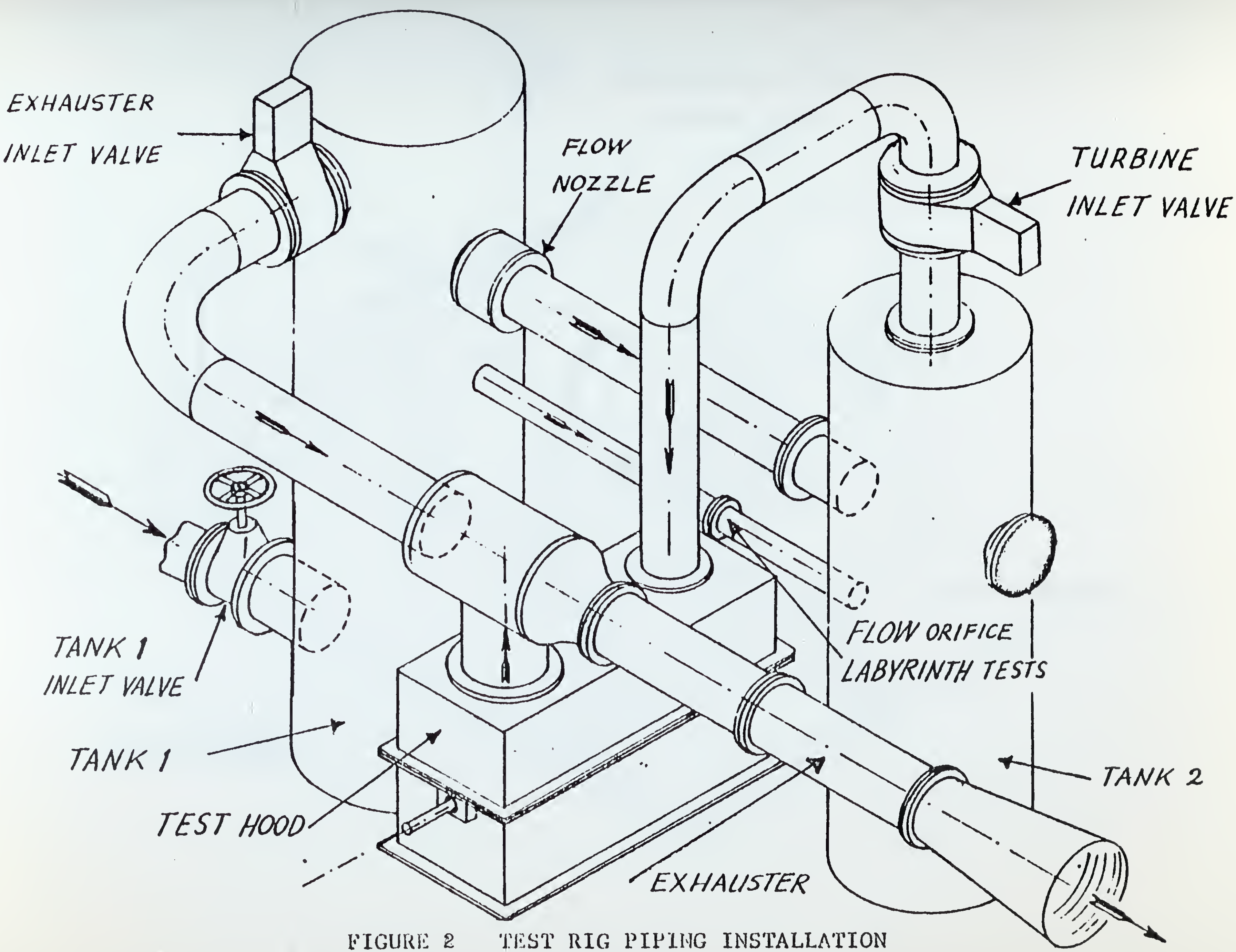


FIGURE 2 TEST RIG PIPING INSTALLATION

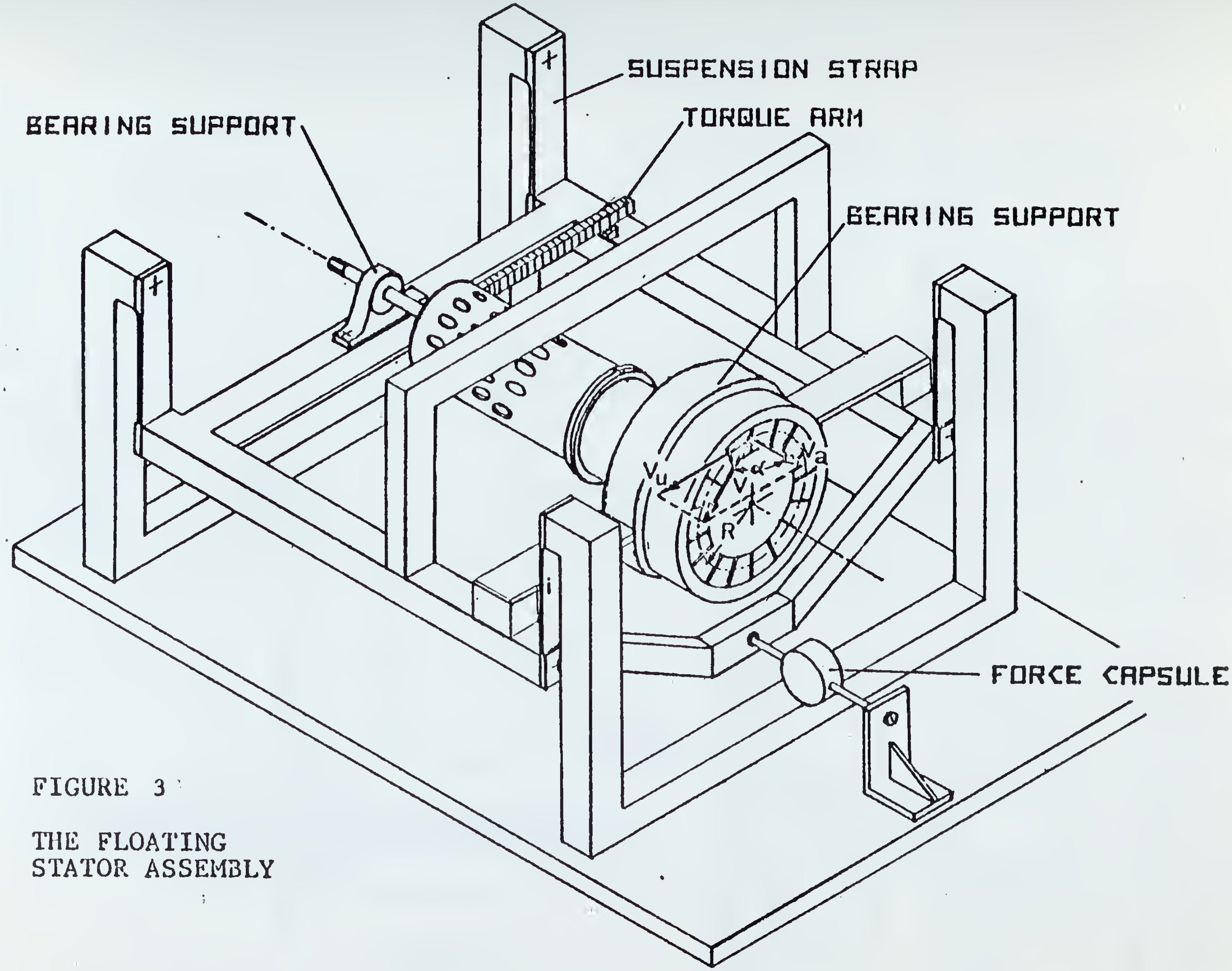


FIGURE 3

THE FLOATING
STATOR ASSEMBLY

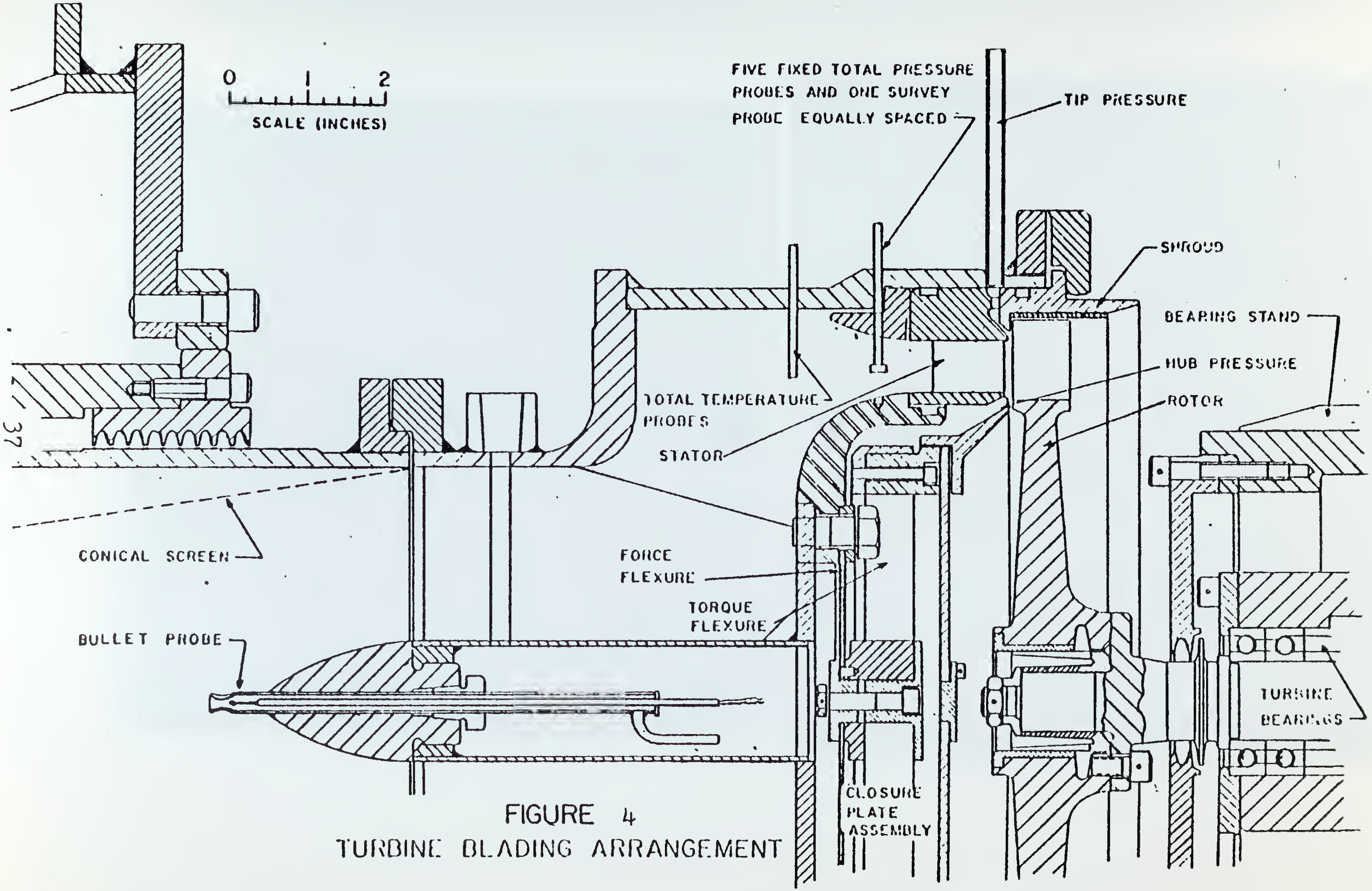


FIGURE 4
TURBINE BLADING ARRANGEMENT

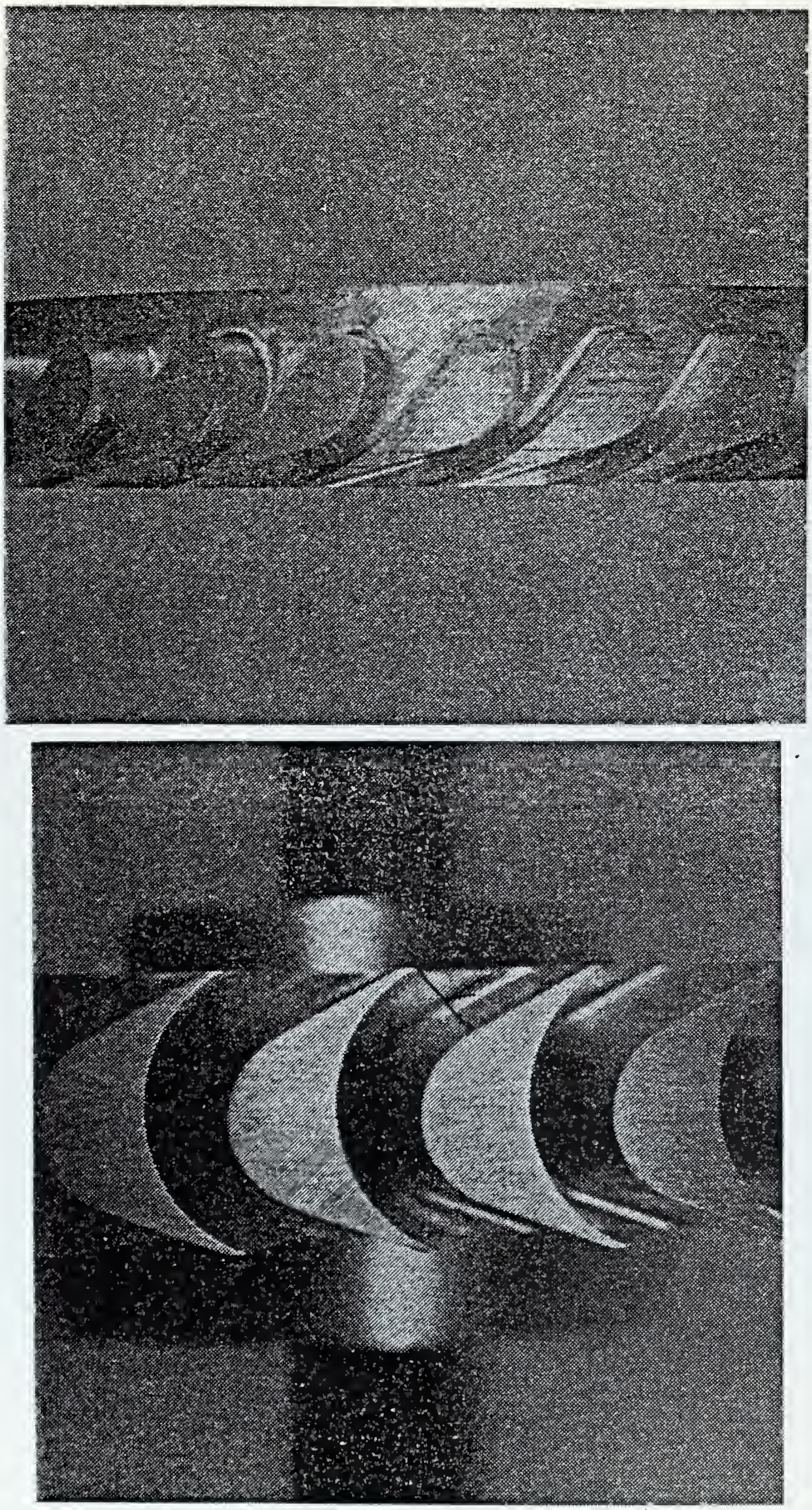


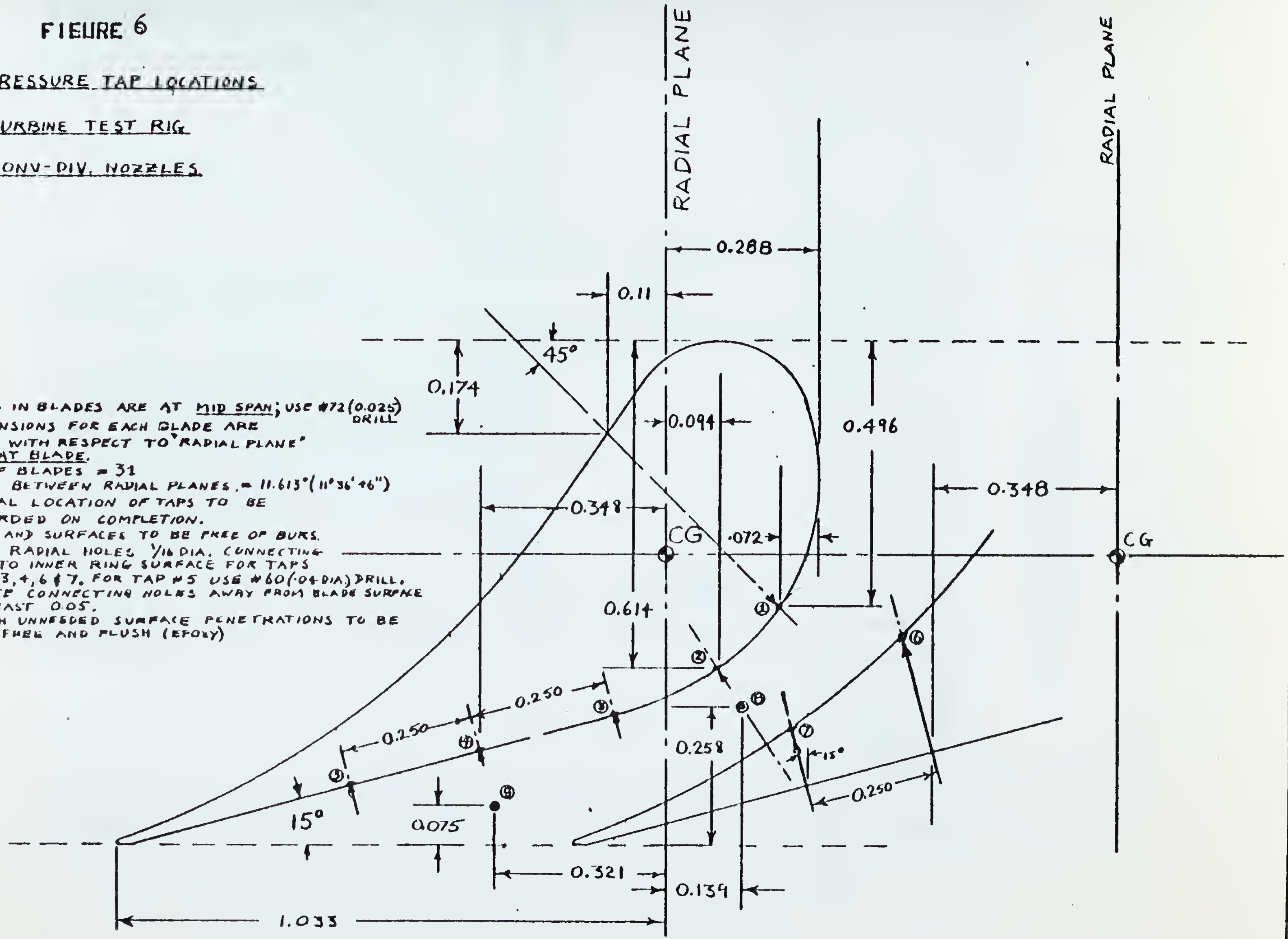
FIGURE 5 TURBINE C

FIGURE 6

PRESSURE TAP LOCATIONS

TURBINE TEST RIG

CONV-DIV. NOZZLES.



NOTE:

1. HOLES IN BLADES ARE AT MID SPAN; USE #72 (0.025) DRILL
2. DIMENSIONS FOR EACH BLADE ARE GIVEN WITH RESPECT TO "RADIAL PLANE" IN THAT BLADE.
   NO. OF BLADES = 31
   ANGLE BETWEEN RADIAL PLANES = 11.613° (11°36'46")
3. ACTUAL LOCATION OF TAPS TO BE RECORDED ON COMPLETION.
4. TAPS AND SURFACES TO BE FREE OF BURS.
5. DRILL RADIAL HOLES 1/16 DIA. CONNECTING TAPS TO INNER RING SURFACE FOR TAPS #1, 2, 3, 4, 6 & 7. FOR TAP #5 USE #60 (.04 DIA.) DRILL. LOCATE CONNECTING HOLES AWAY FROM BLADE SURFACE AT LEAST 0.05.
6. FINISH UNNEEDED SURFACE PENETRATIONS TO BE LEAK-FREE AND FLUSH (EPOXY)

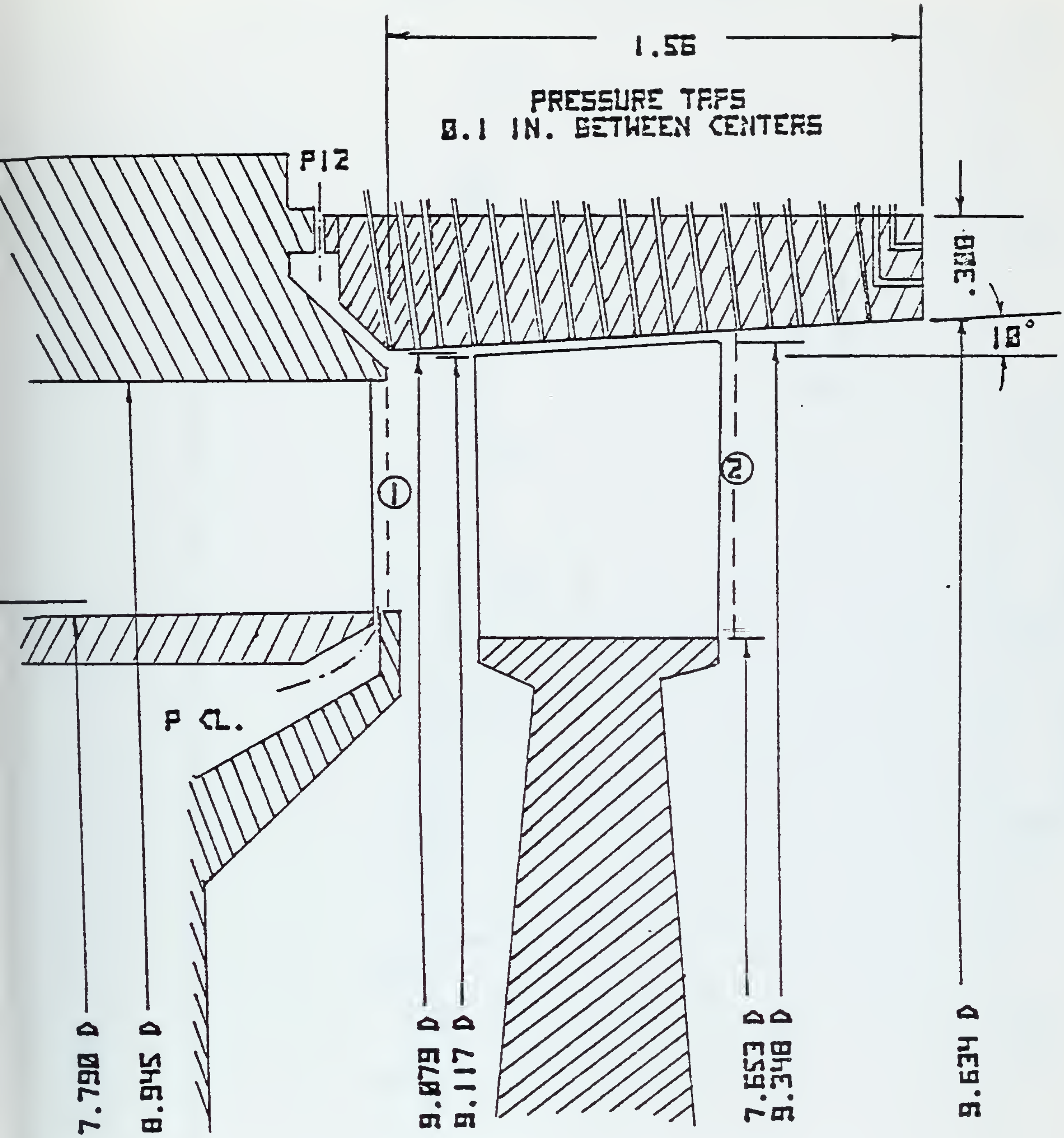


FIGURE 7 TURBINE TEST RIG GEOMETRY FOR TURBINE CONFIGURATION C

FIGURE 8 ILLUSTRATION OF SMOOTHING USING FIRST DEGREE POLYNOMIAL

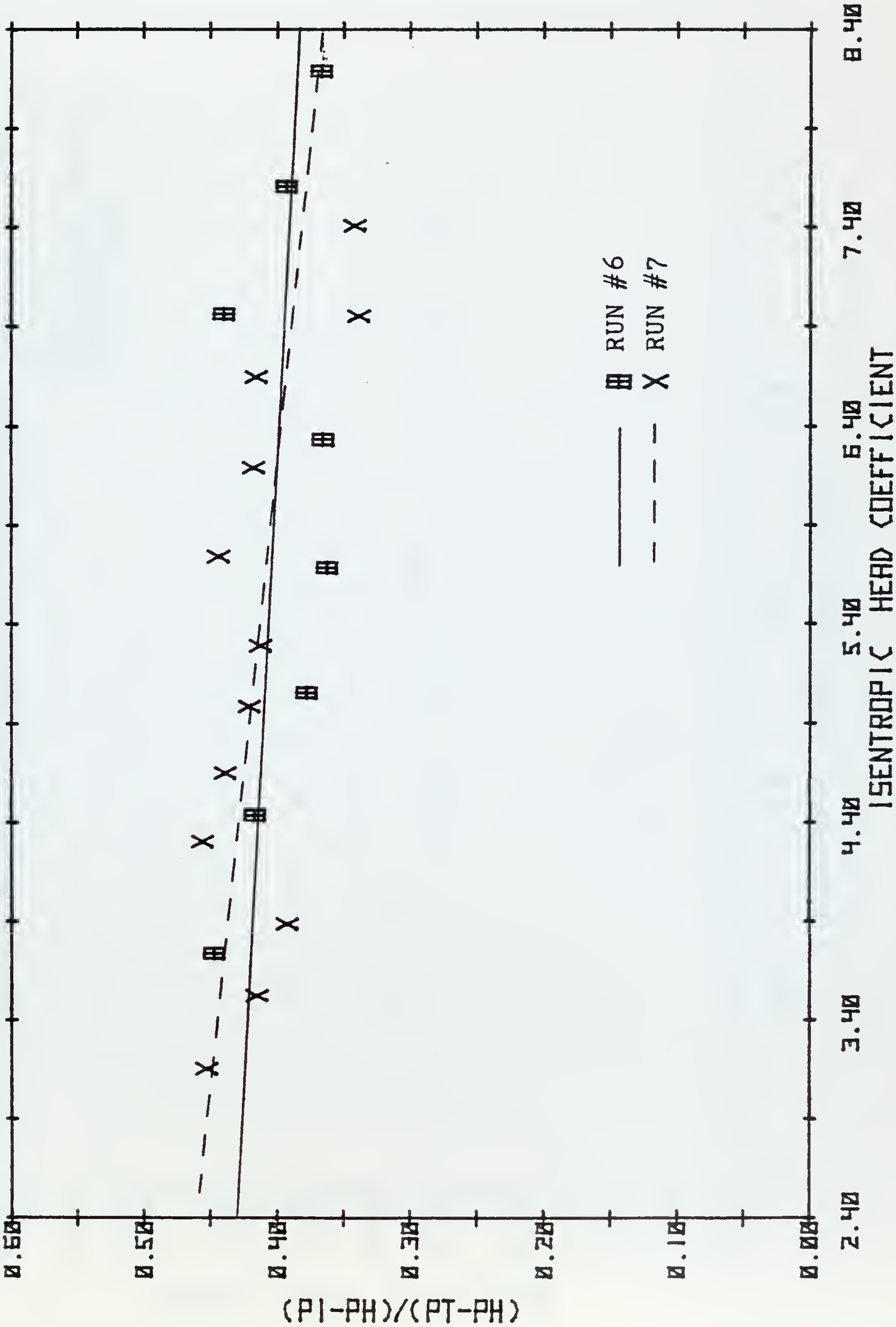


FIGURE 9 STATOR LOSS VS. ISENTROPIC HEAD COEFFICIENT, BEFORE SMOOTHING

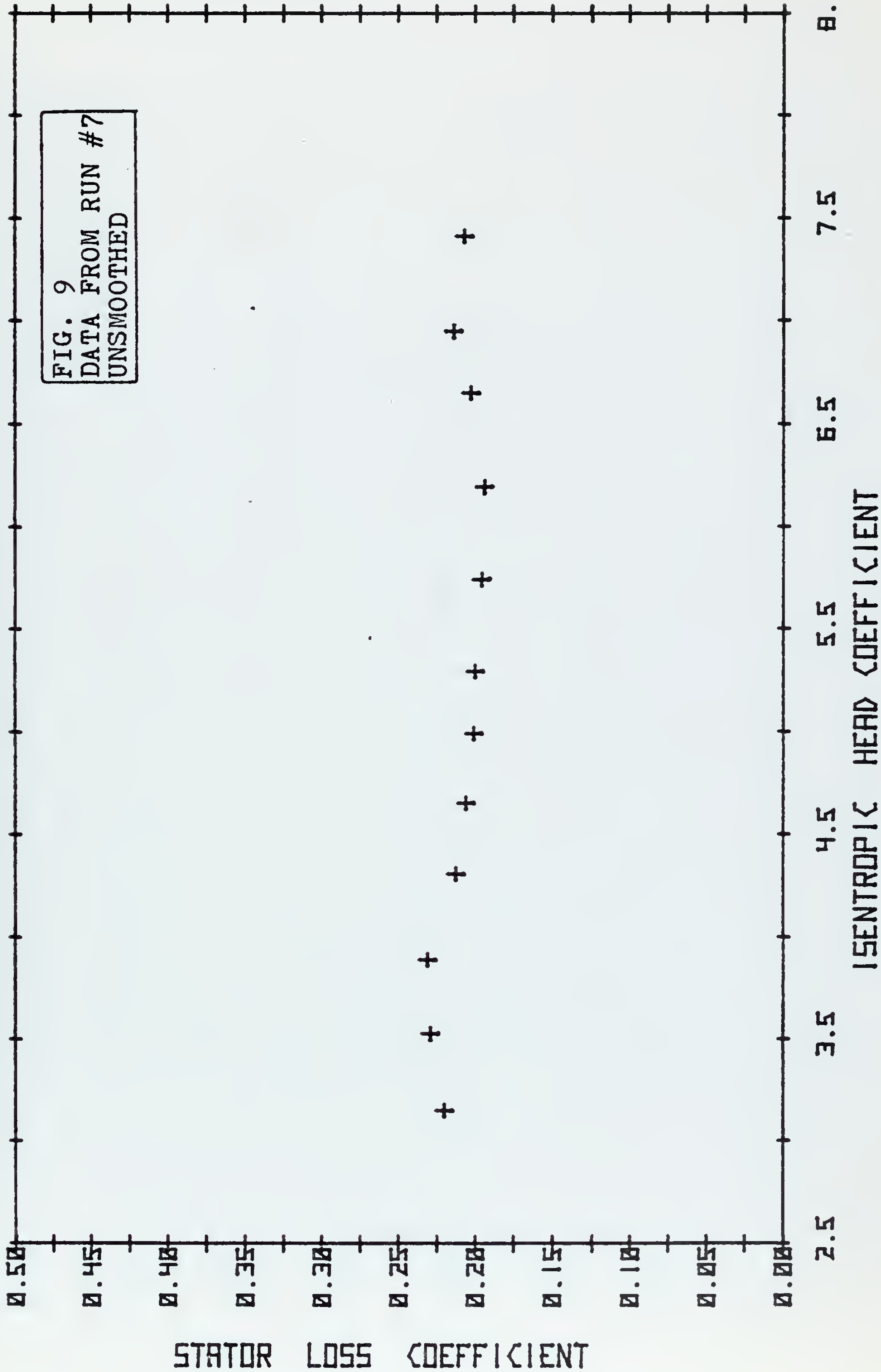


FIGURE 10 STATOR LOSS VS. ISENTROPIC HEAD COEFFICIENT, AFTER SMOOTHING

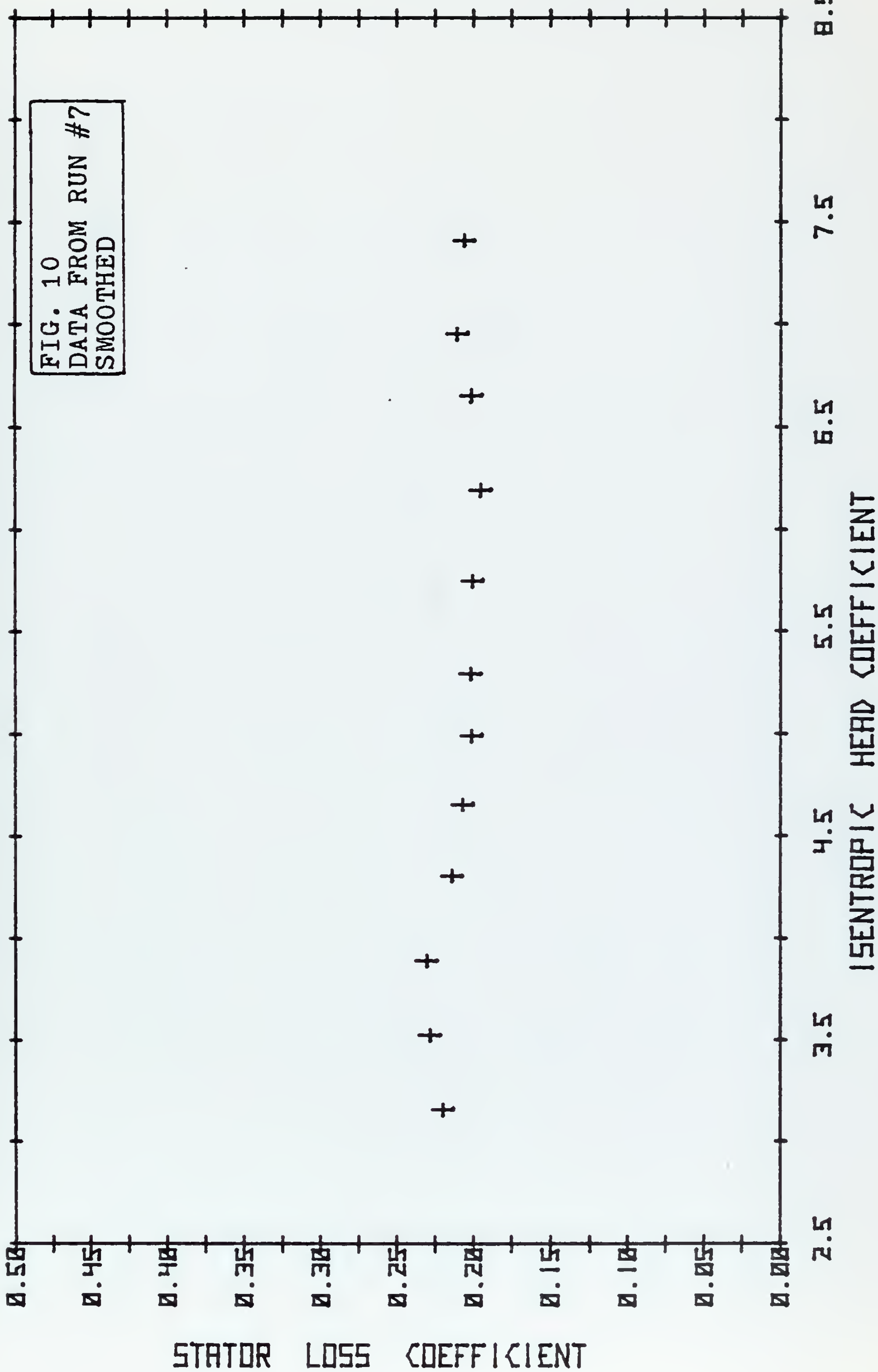


FIGURE 11 ROTOR LOSS VS. ISENTROPIC HEAD COEFFICIENT, BEFORE SMOOTHING

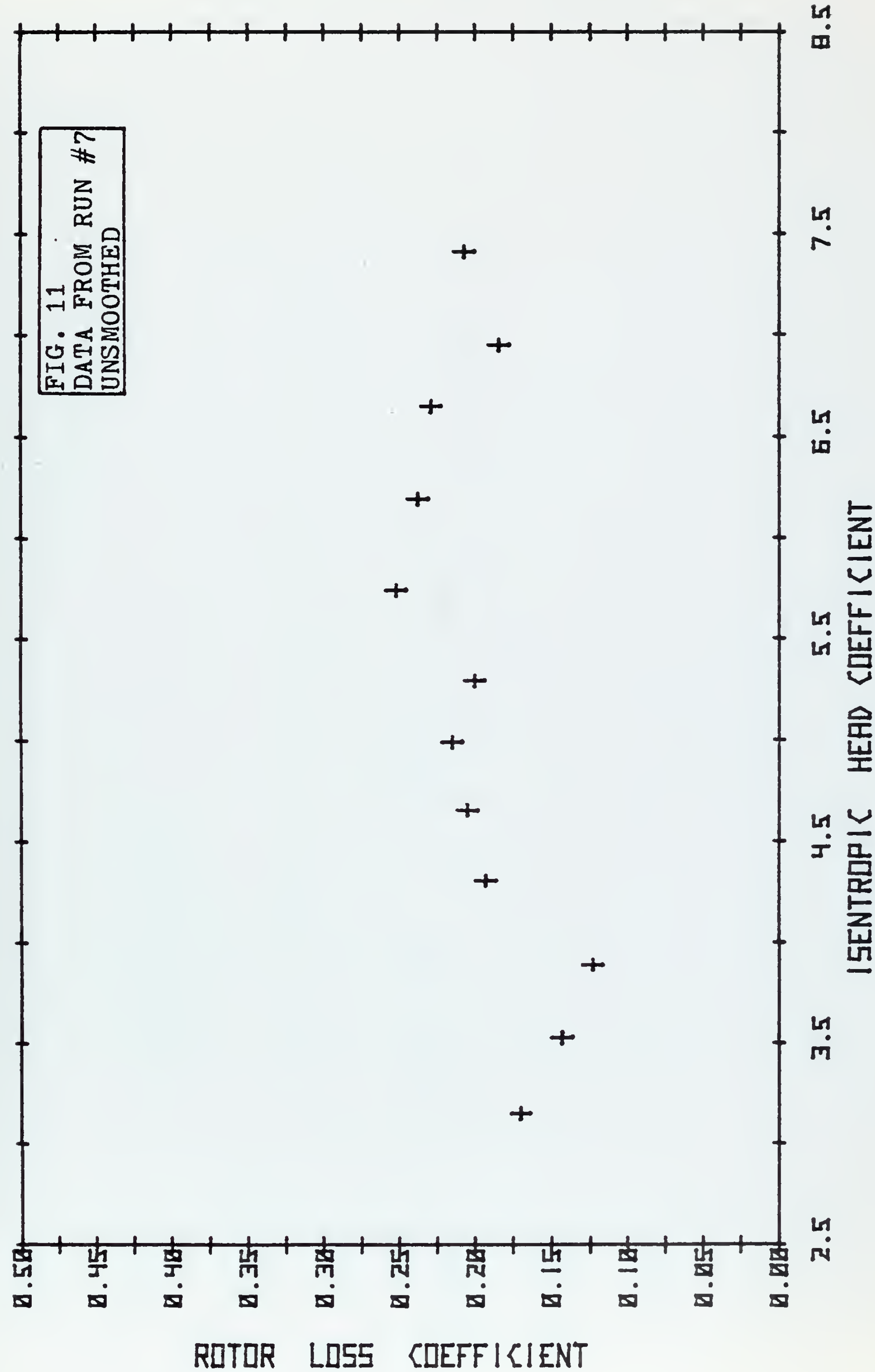


FIGURE 12 ROTOR LOSS VS. ISENTROPIC HEAD
COEFFICIENT, AFTER SMOOTHING

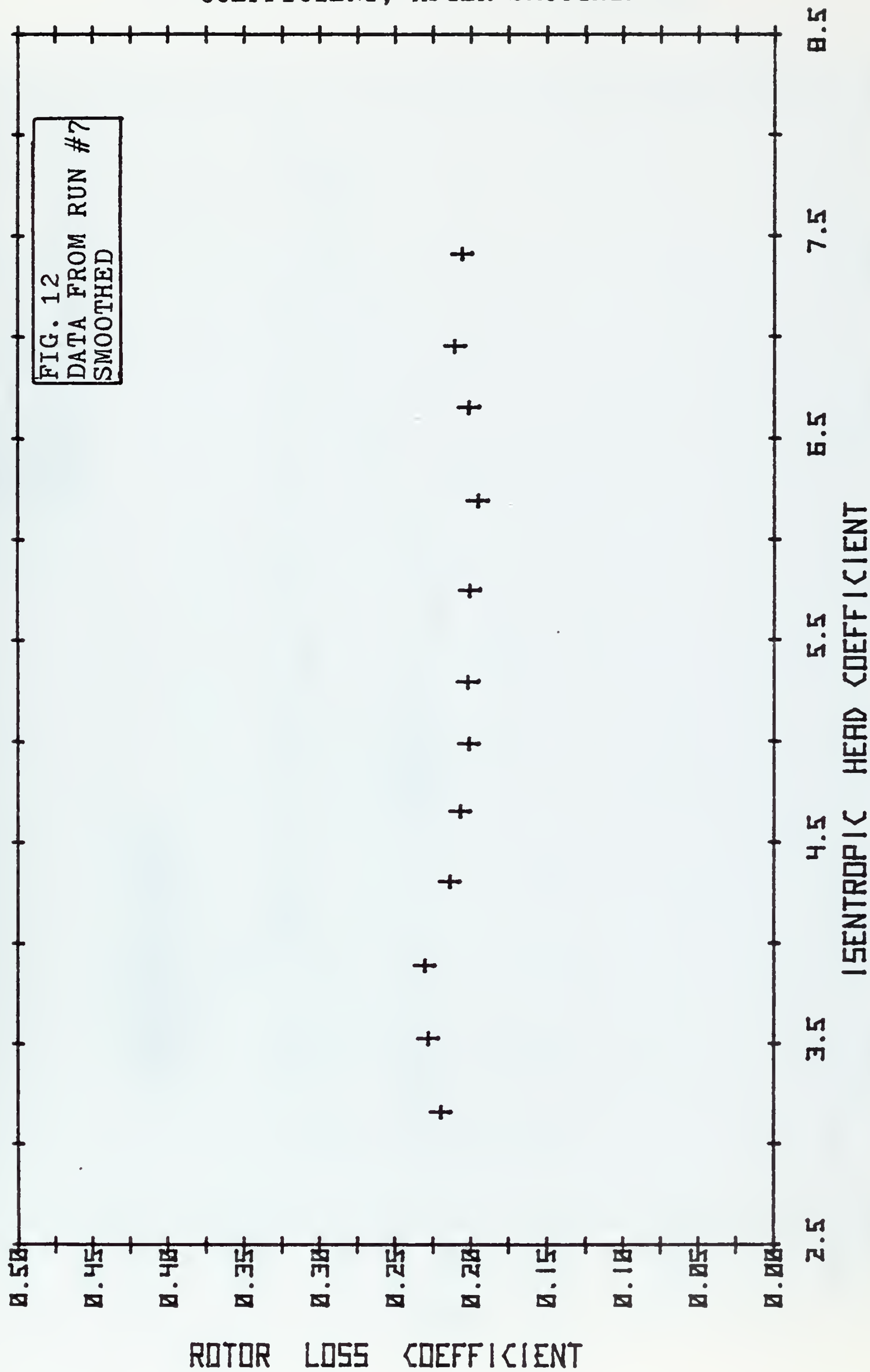


FIGURE 13 PREDICTED PERFORMANCE RANGE,
RPM=15,000, INPUT ROTOR LOSS=.2514

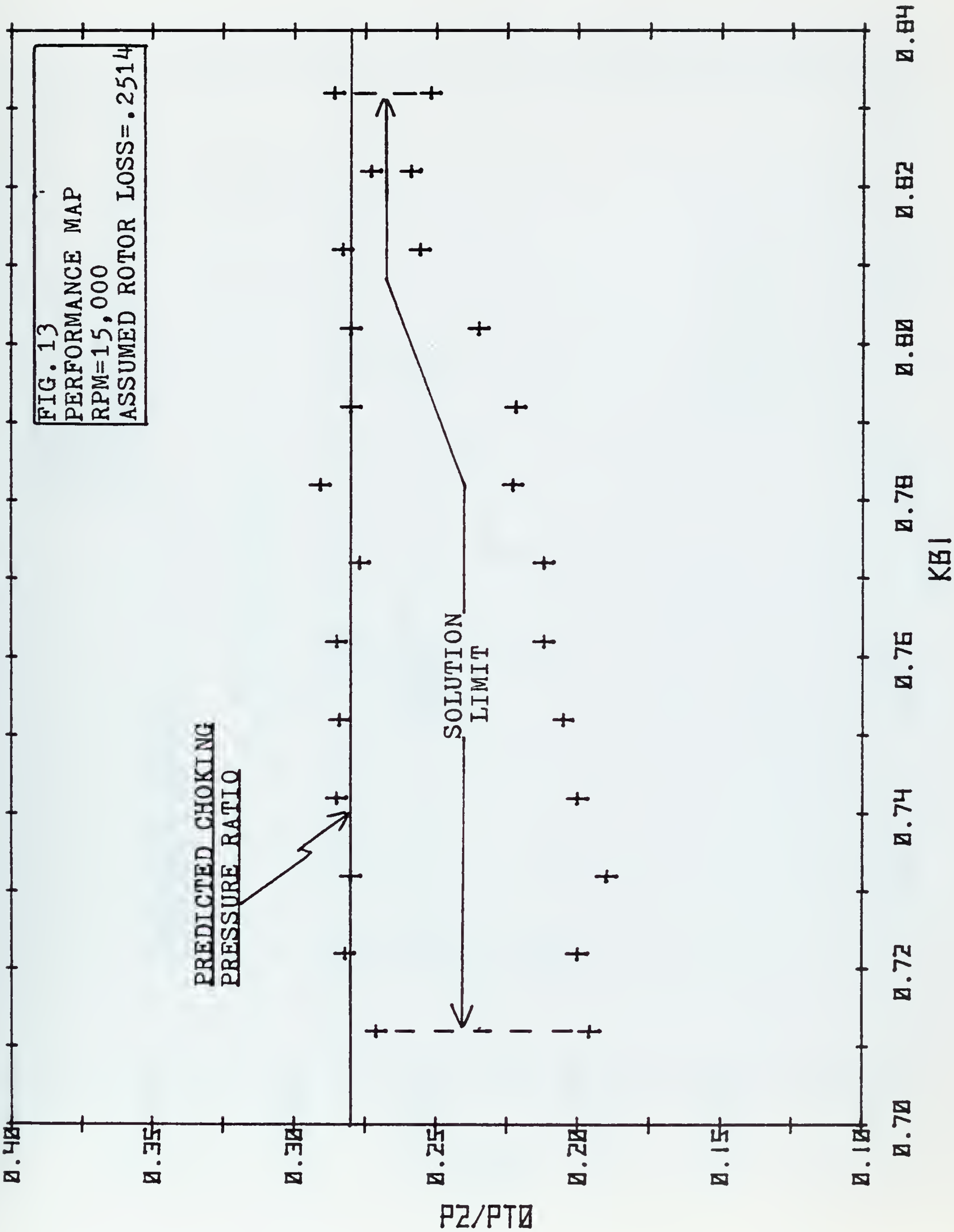


FIGURE 14 PREDICTED PERFORMANCE RANGE,
RPM=18,000, INPUT ROTOR LOSS=.2514

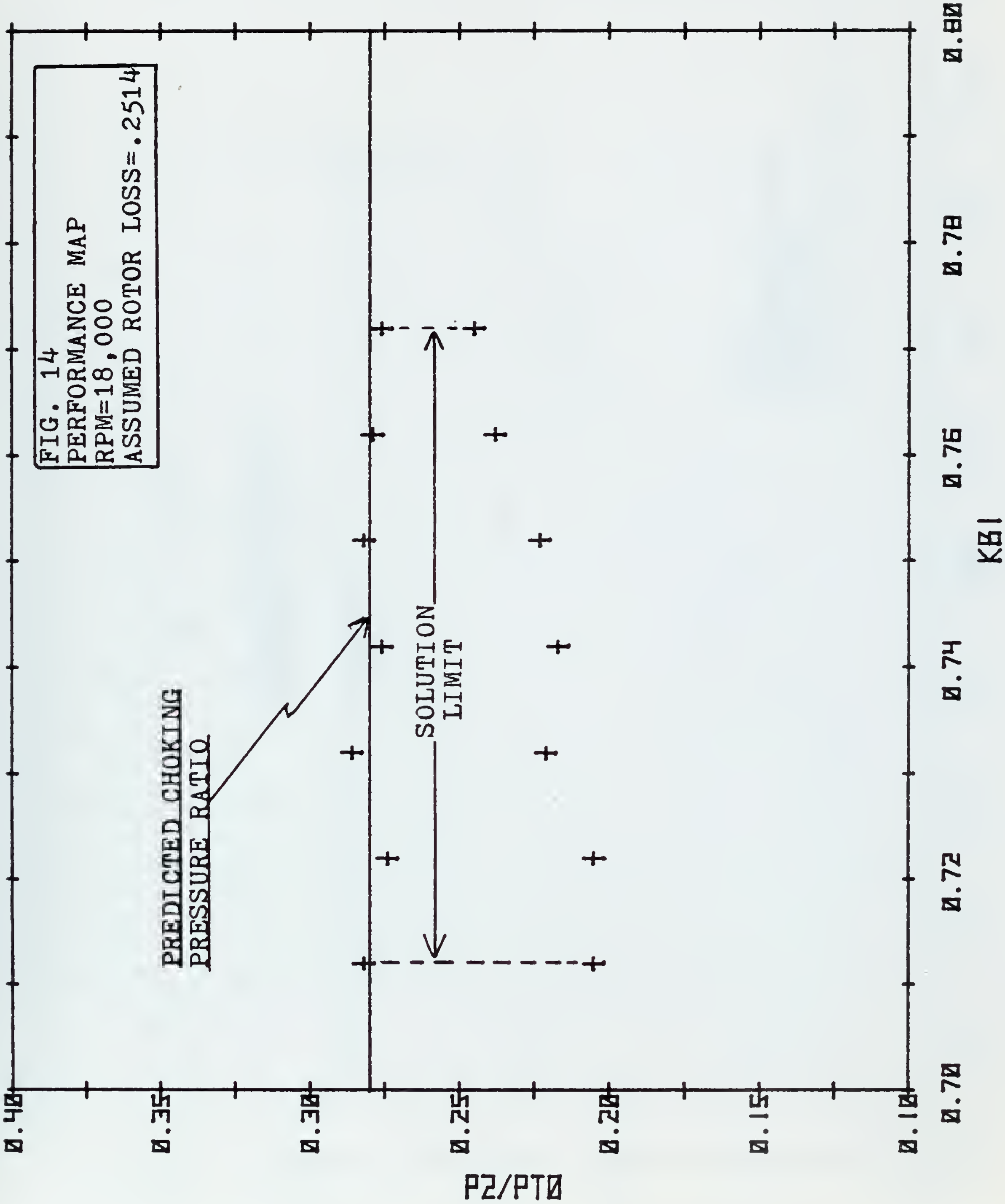


FIGURE 15 PREDICTED PERFORMANCE RANGE,
RPM=18,000, INPUT ROTOR LOSS=.1

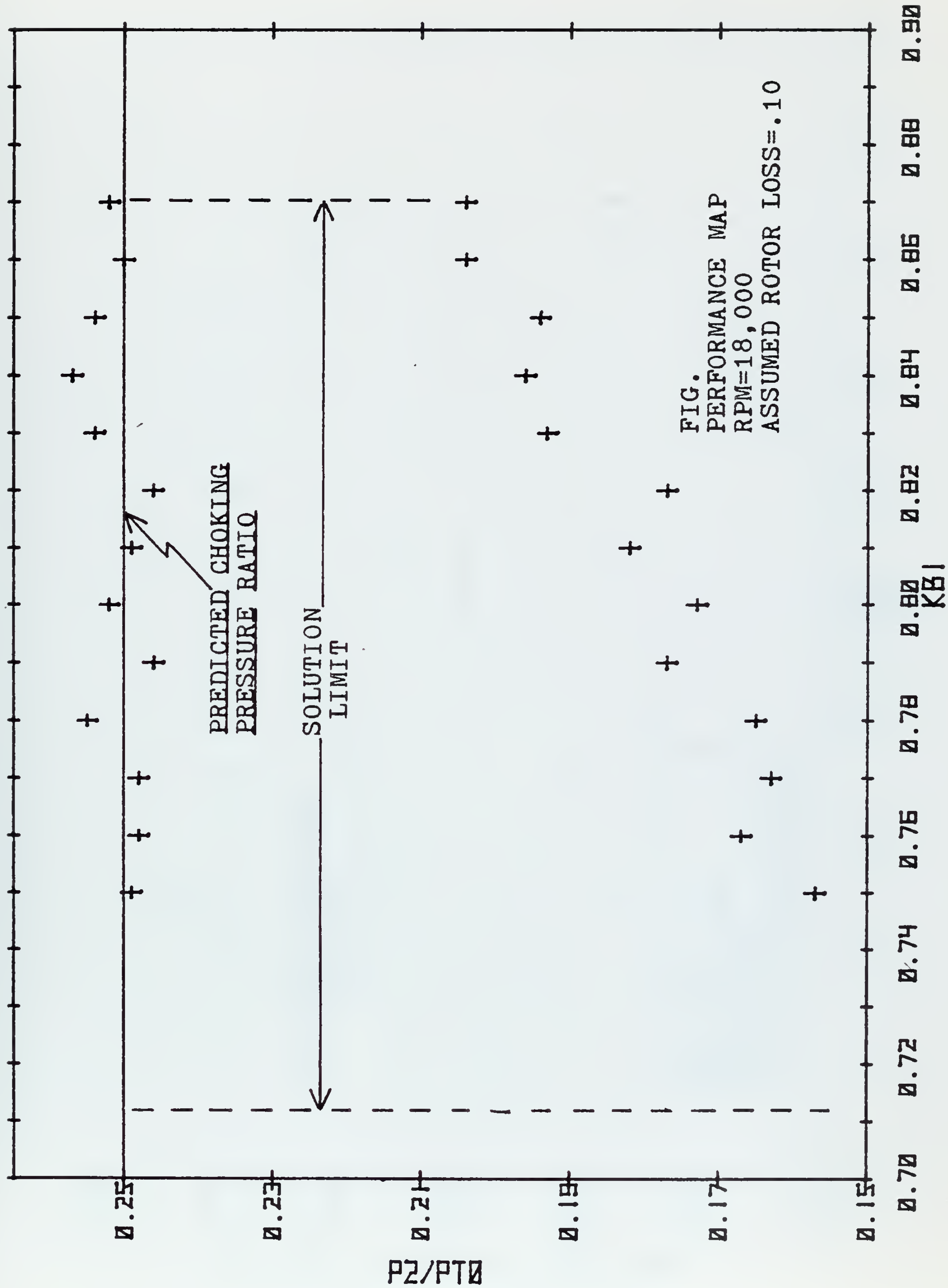


FIGURE 16 PREDICTED REFERRED HORSEPOWER VS. K_{b1}

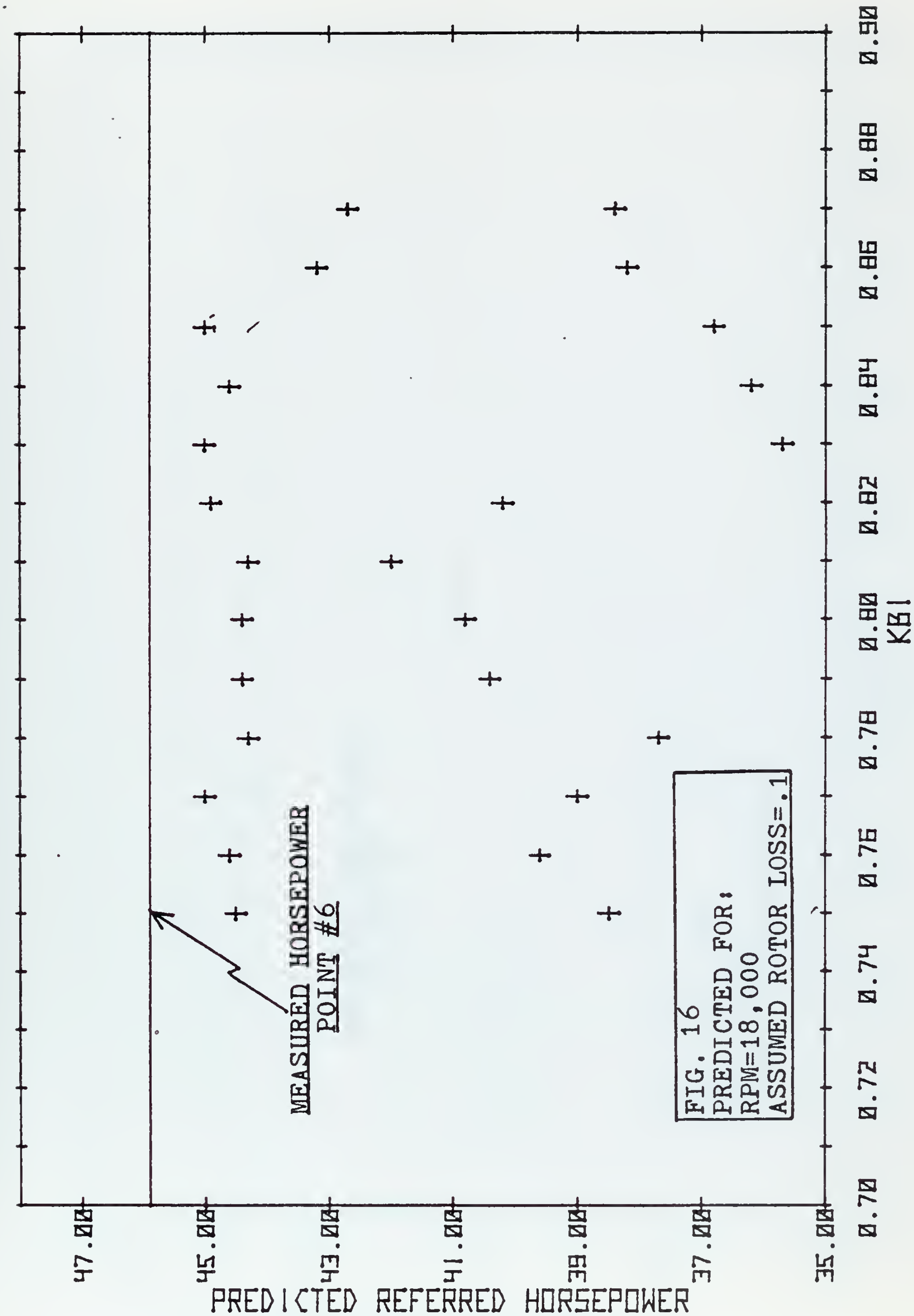


FIGURE 17 PREDICTED EFFICIENCY VS. K_{b1}

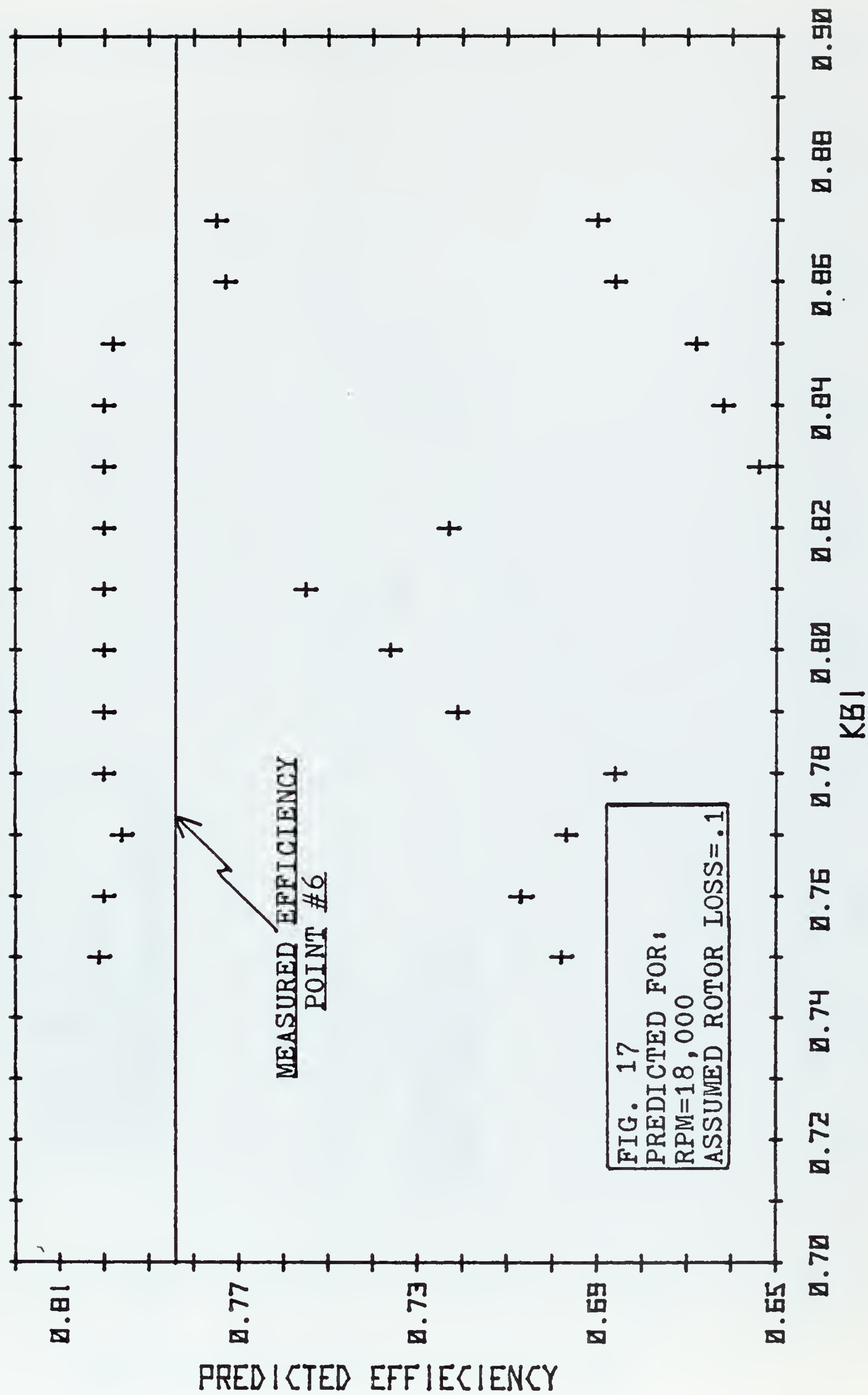


FIGURE 18 PREDICTED STATOR LOSS VS. K_{b1}

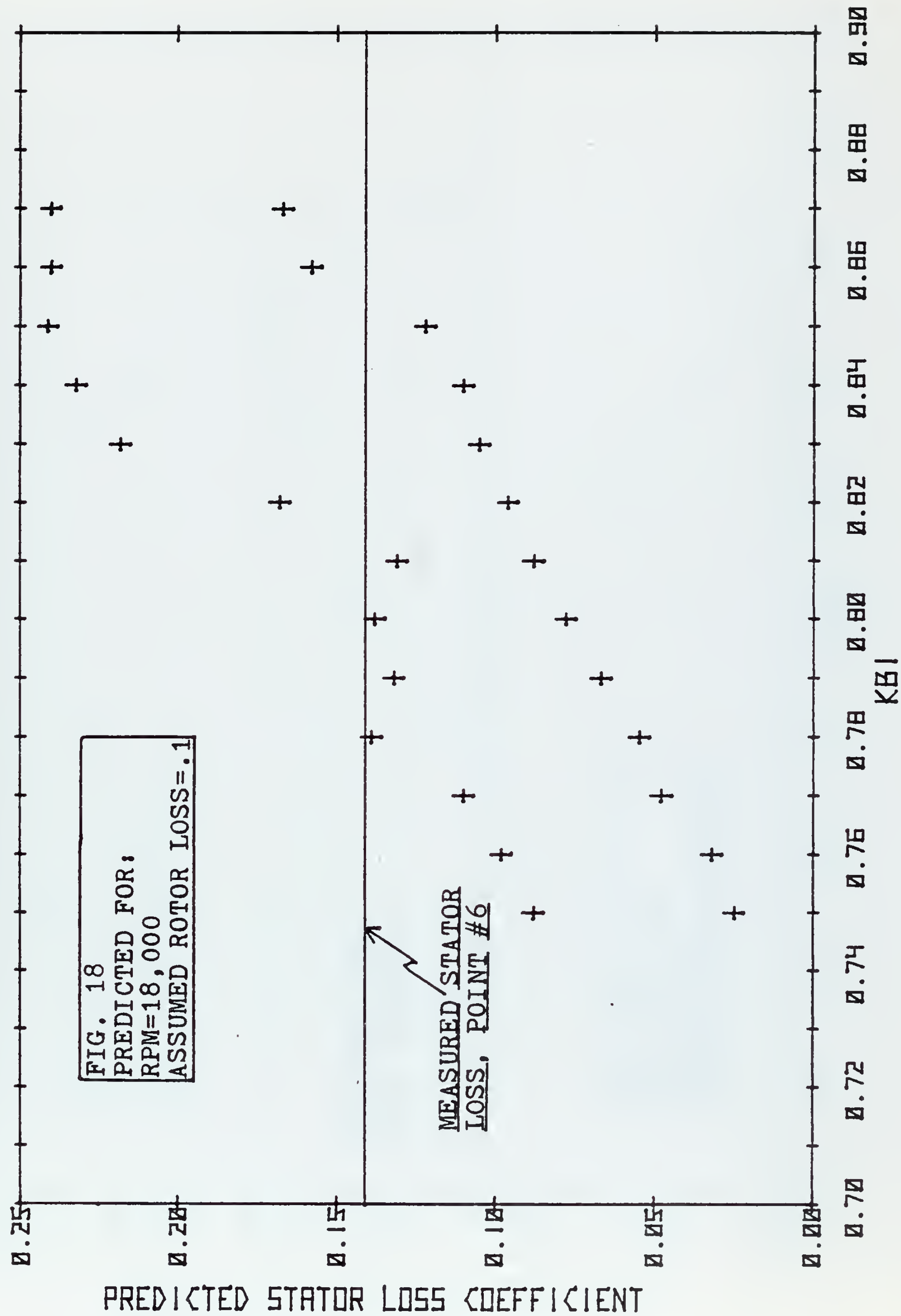


FIGURE 19 PREDICTED ROTOR LOSS VS. K_{b1}

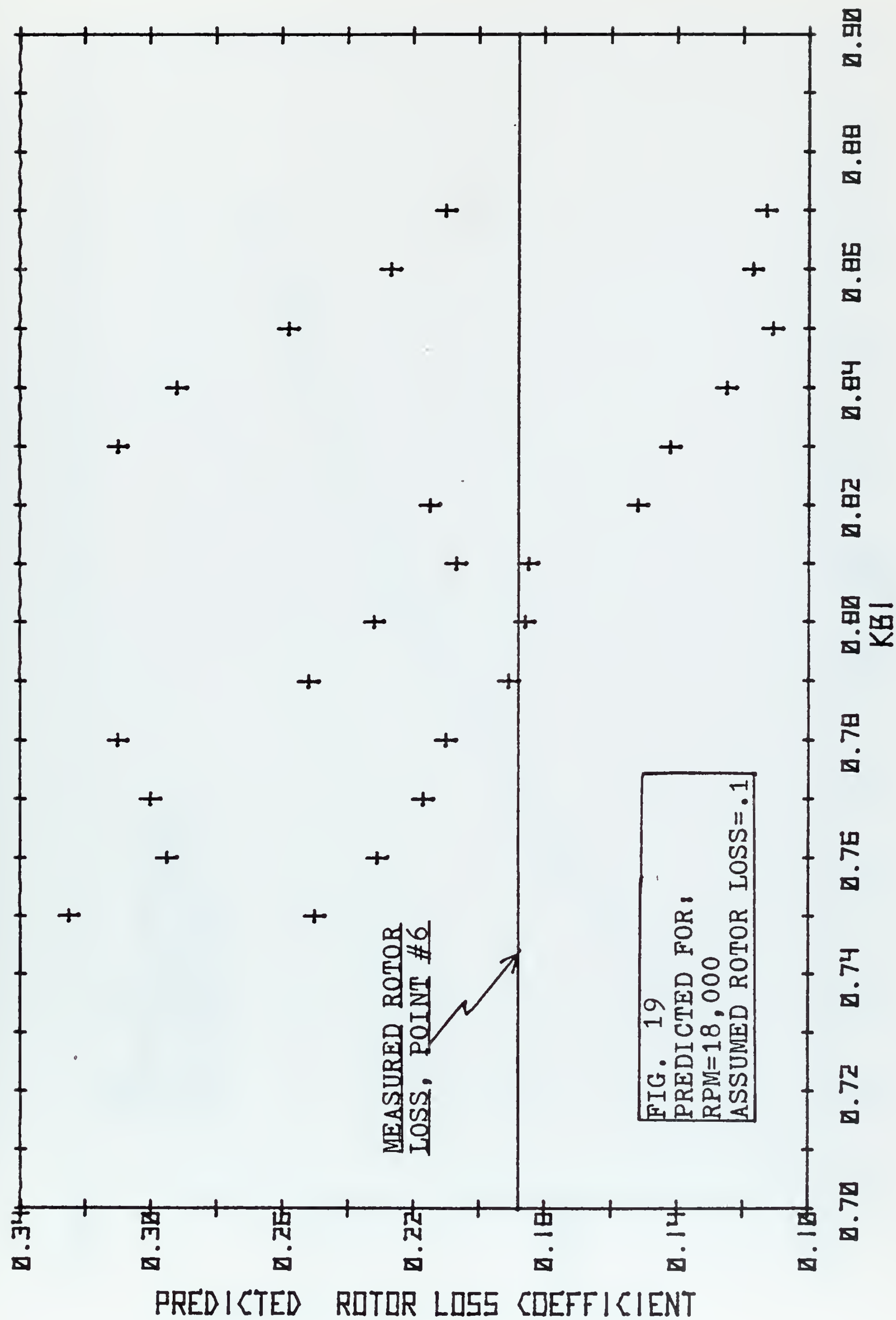


FIGURE 20 REFERRED HORSEPOWER VS. PRESSURE RATIO

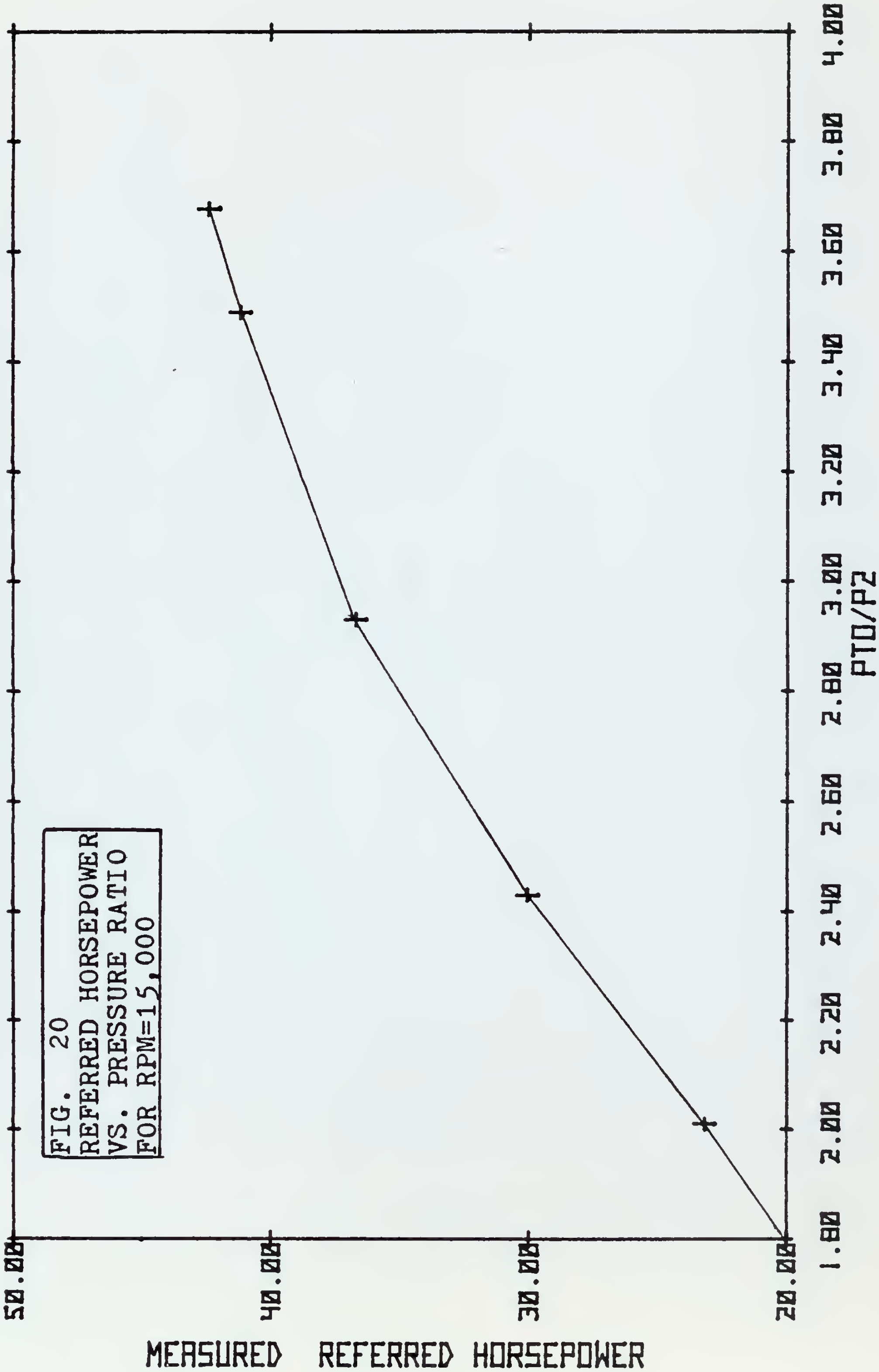


FIGURE 21 REFERRED HORSEPOWER VS. PRESSURE RATIO

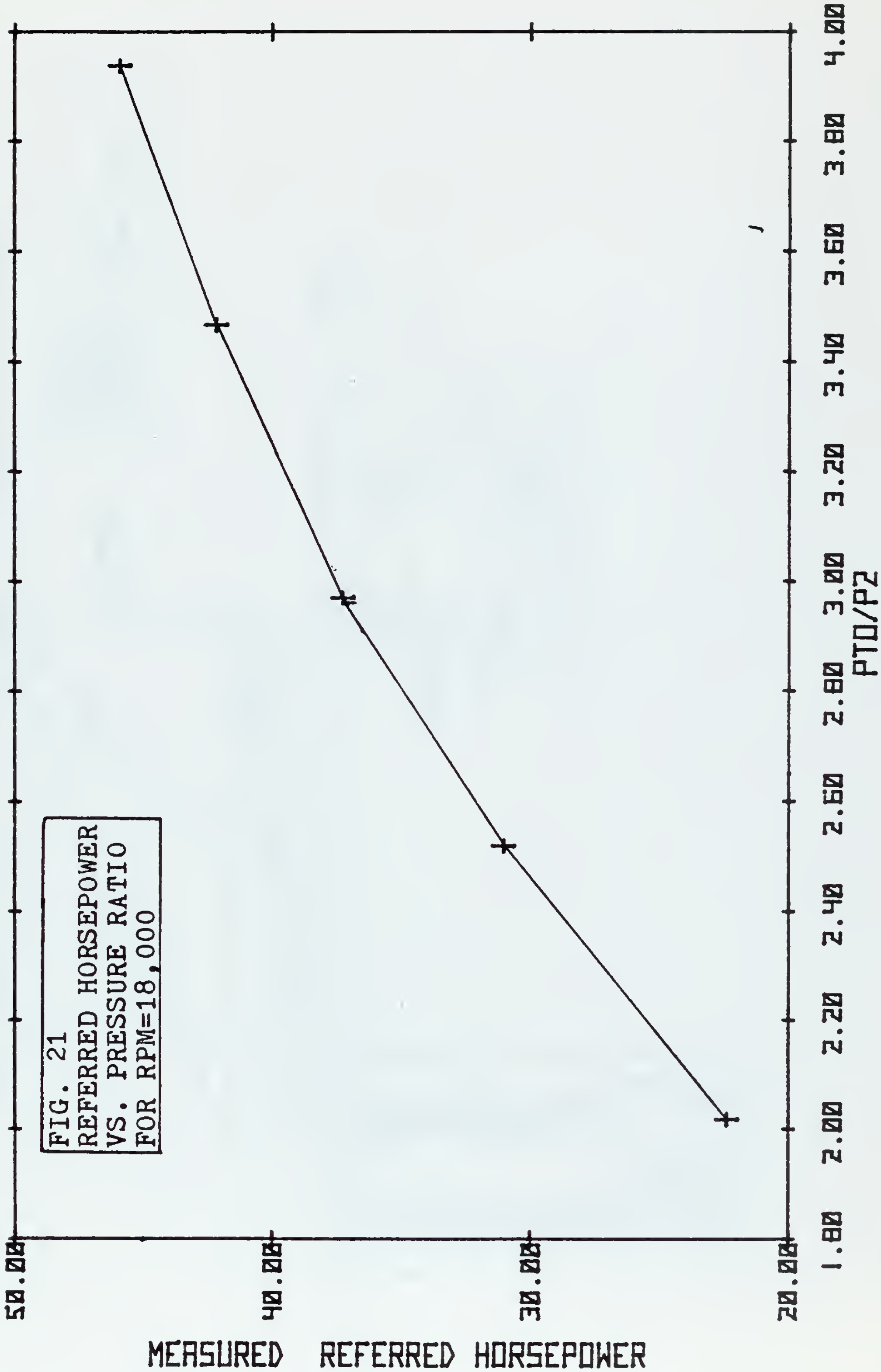


FIGURE 22 VELOCITY DIAGRAM FOR TURBINE TEST

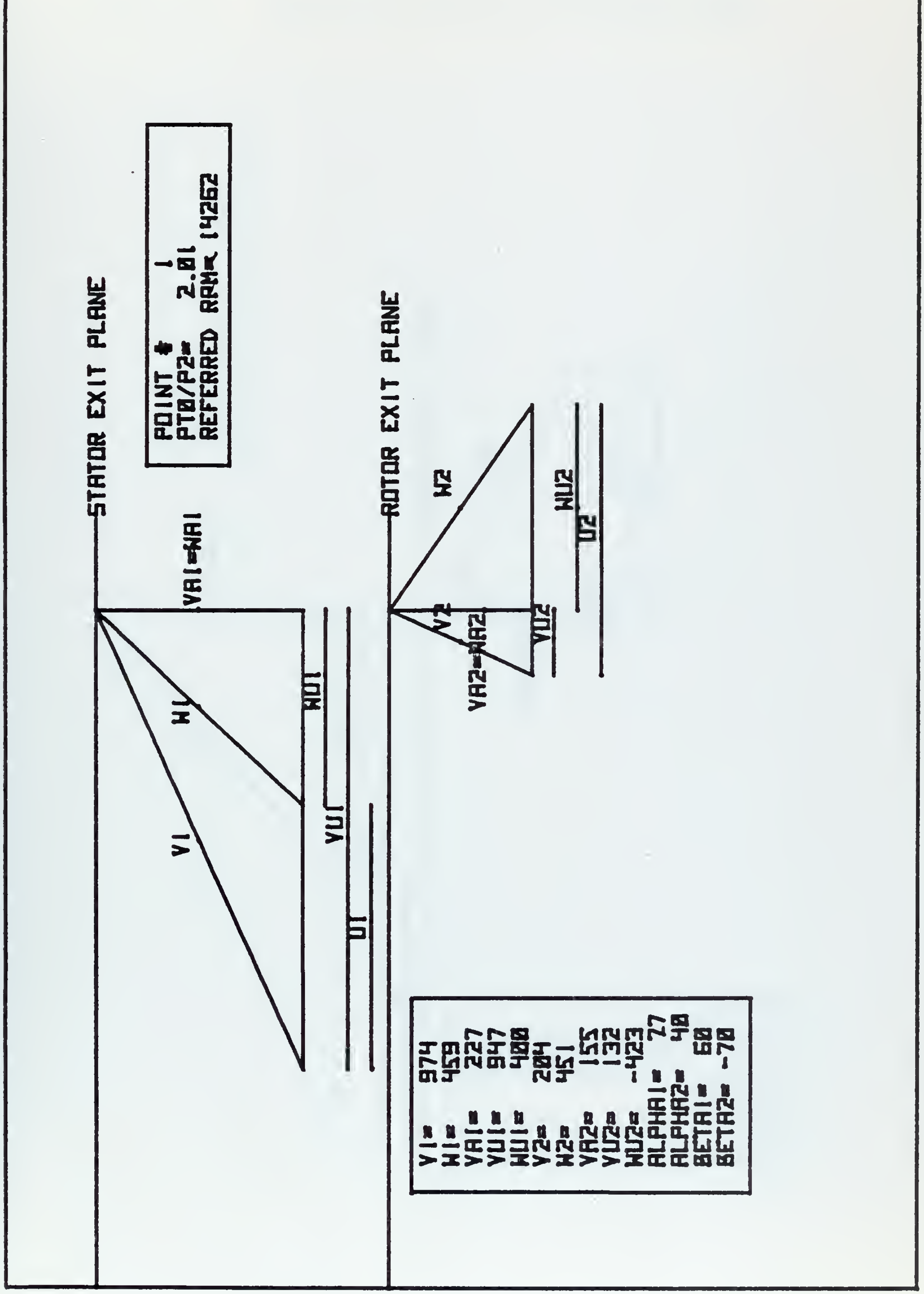


FIGURE 23 VELOCITY DIAGRAM FOR TURBINE TEST

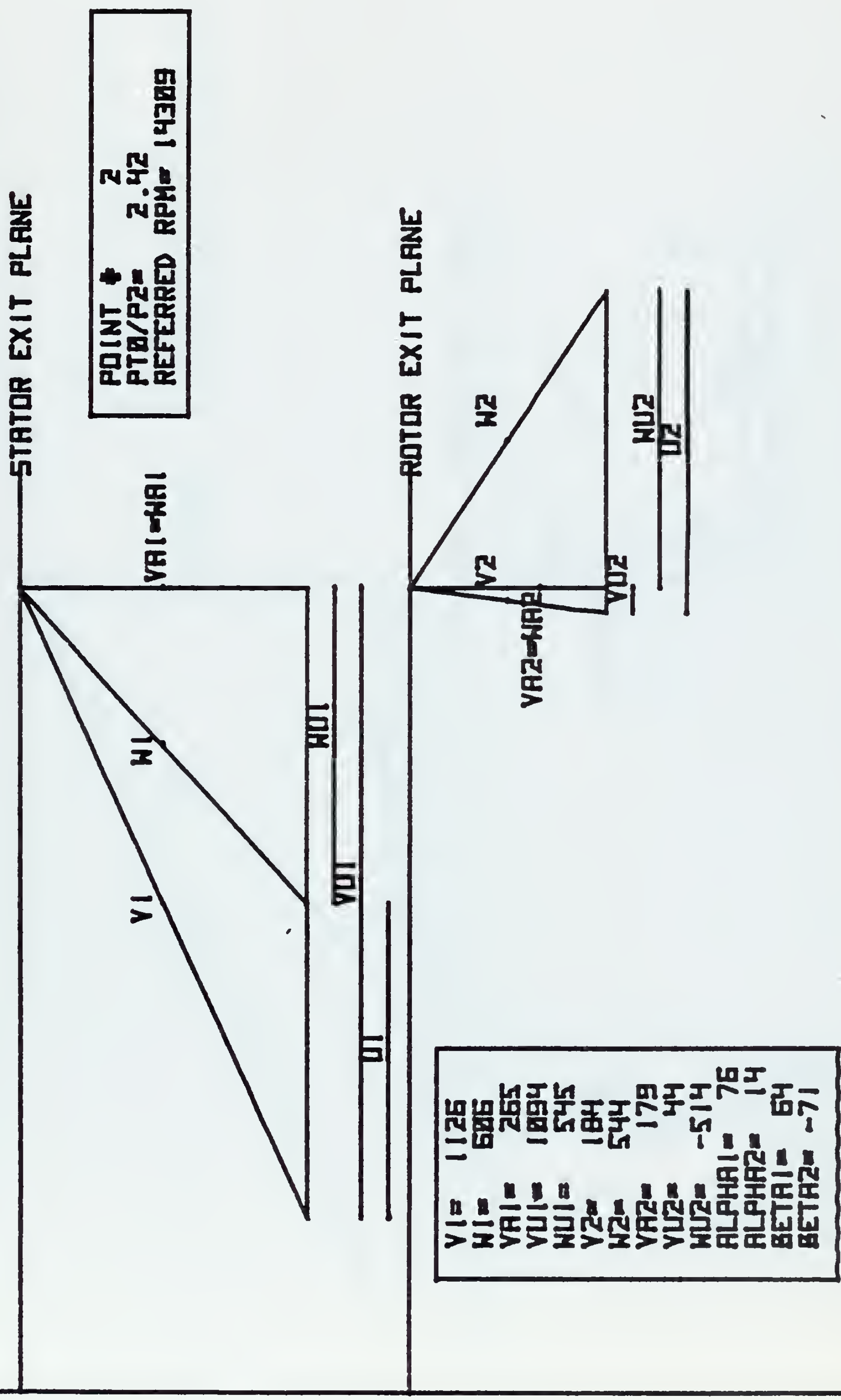


FIGURE 24 VELOCITY DIAGRAM FOR TURBINE TEST

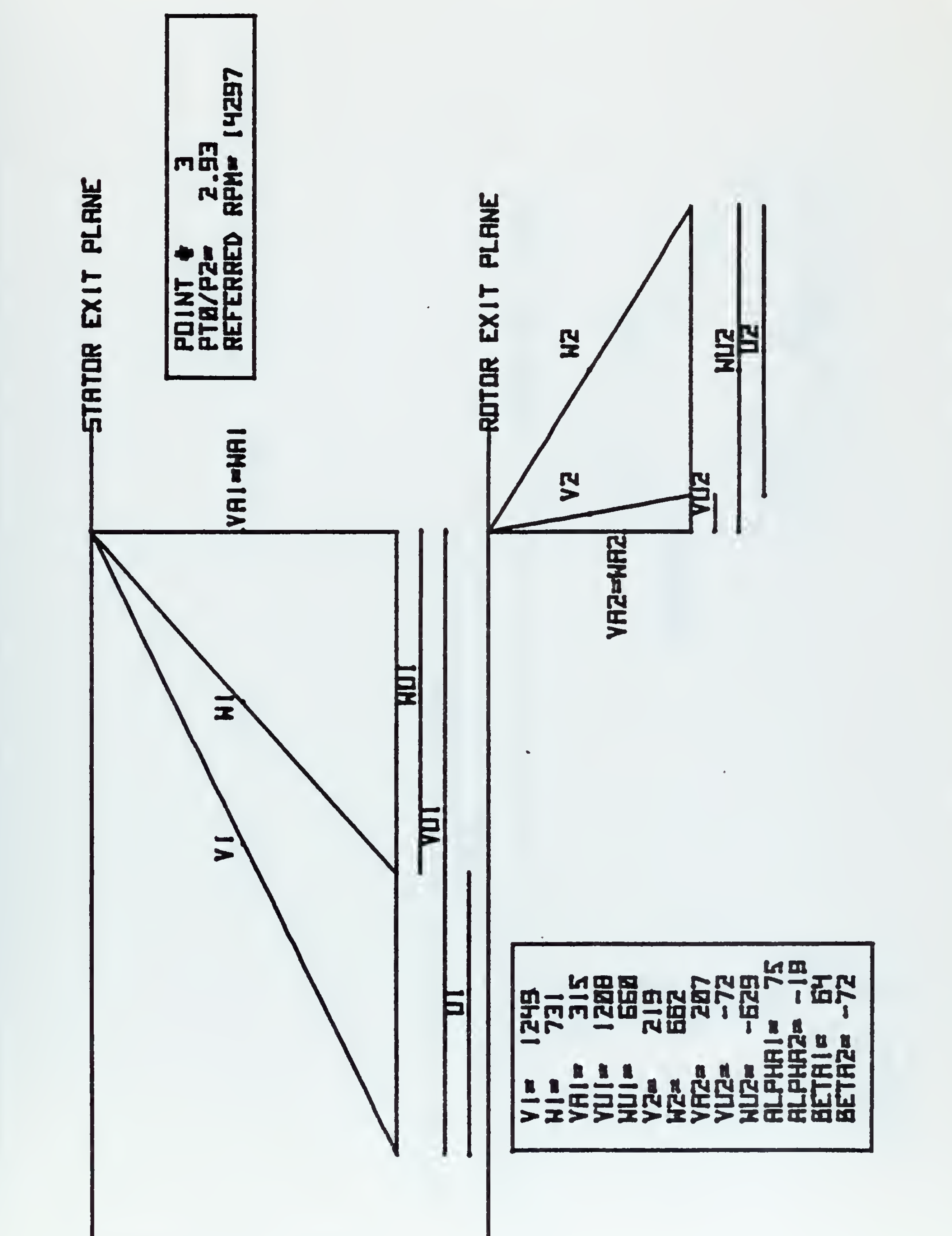


FIGURE 25 VELOCITY DIAGRAM FOR TURBINE TEST

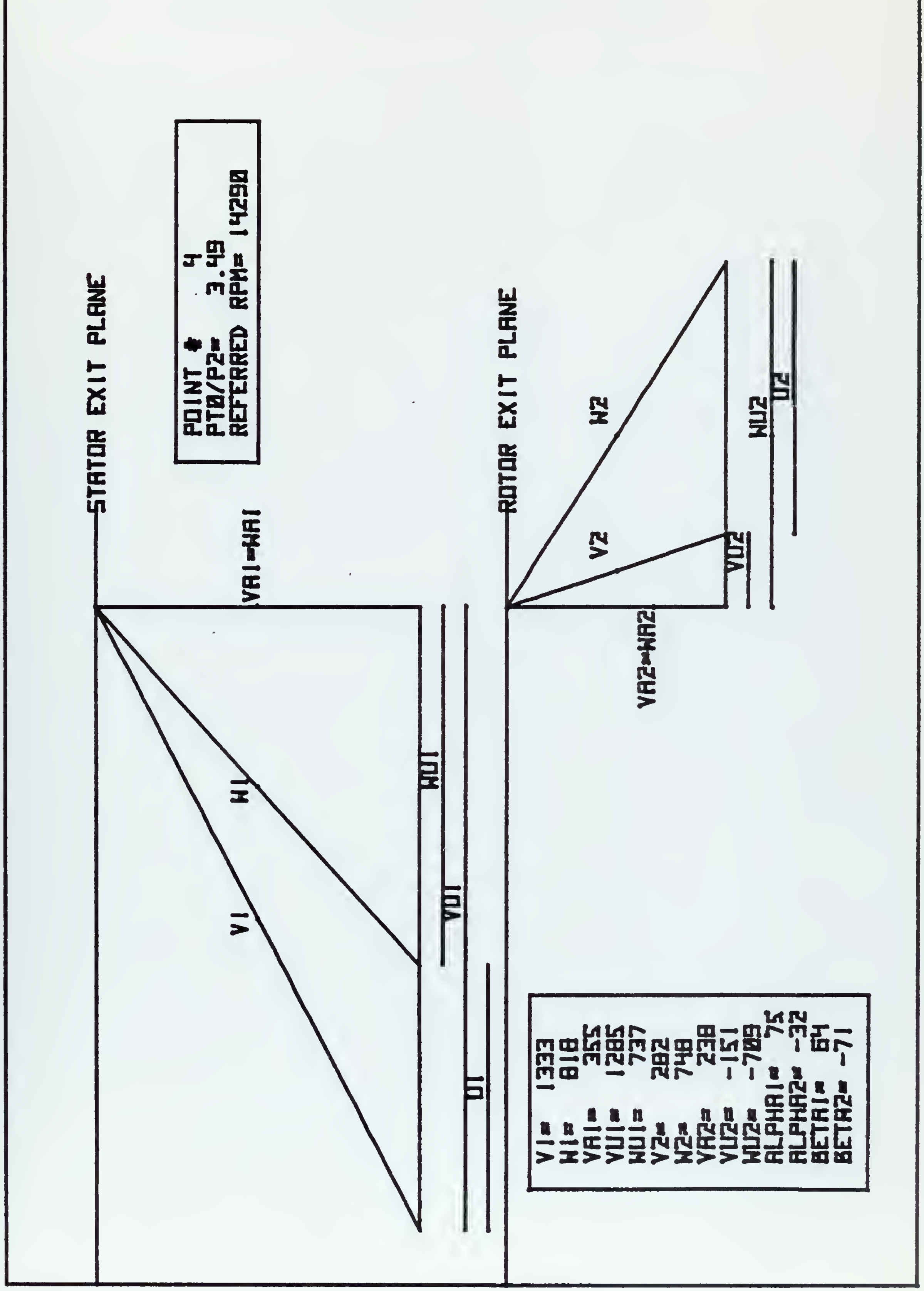


FIGURE 26 VELOCITY DIAGRAM FOR TURBINE TEST

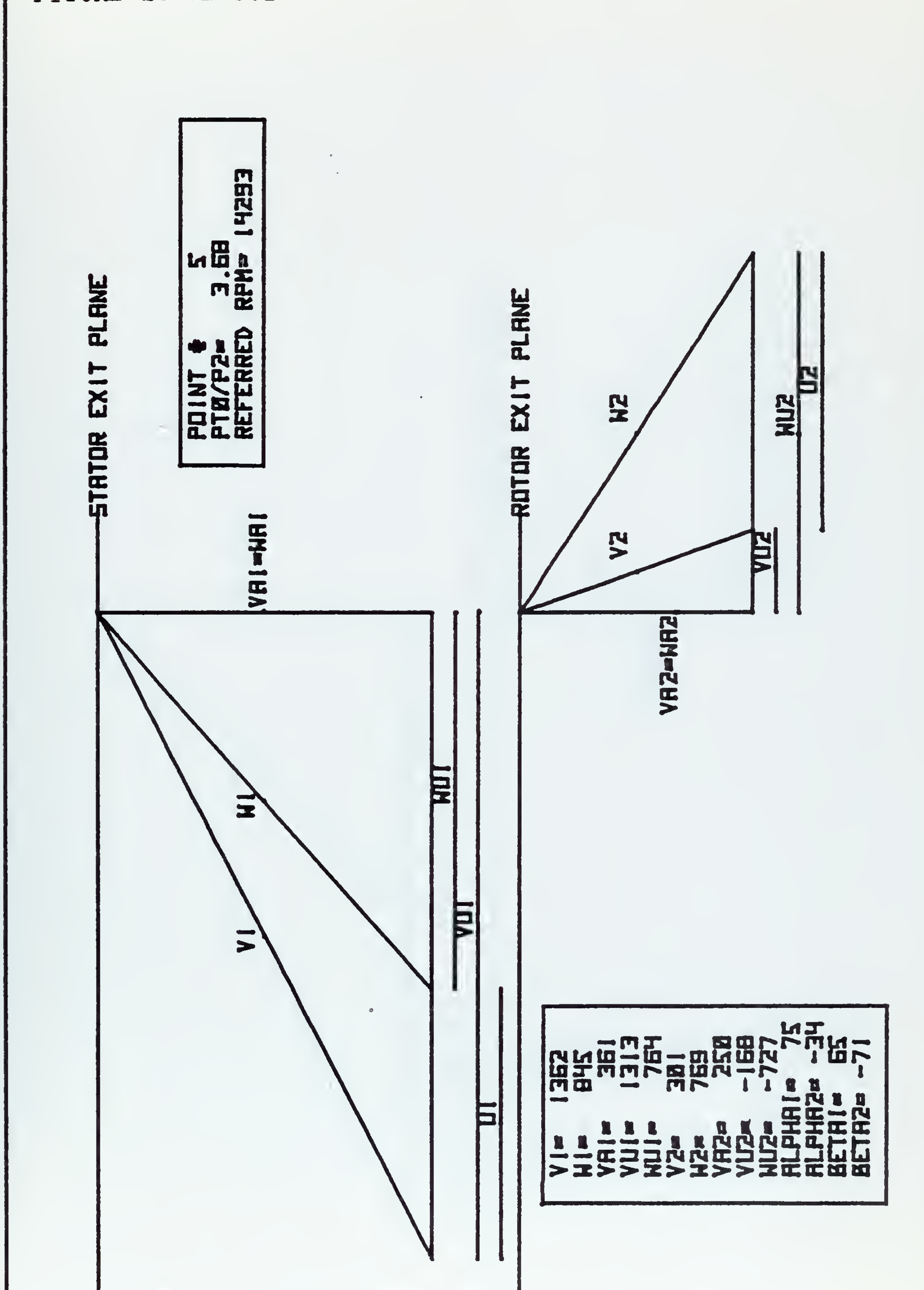


FIGURE 27 VELOCITY DIAGRAM FOR TURBINE TEST

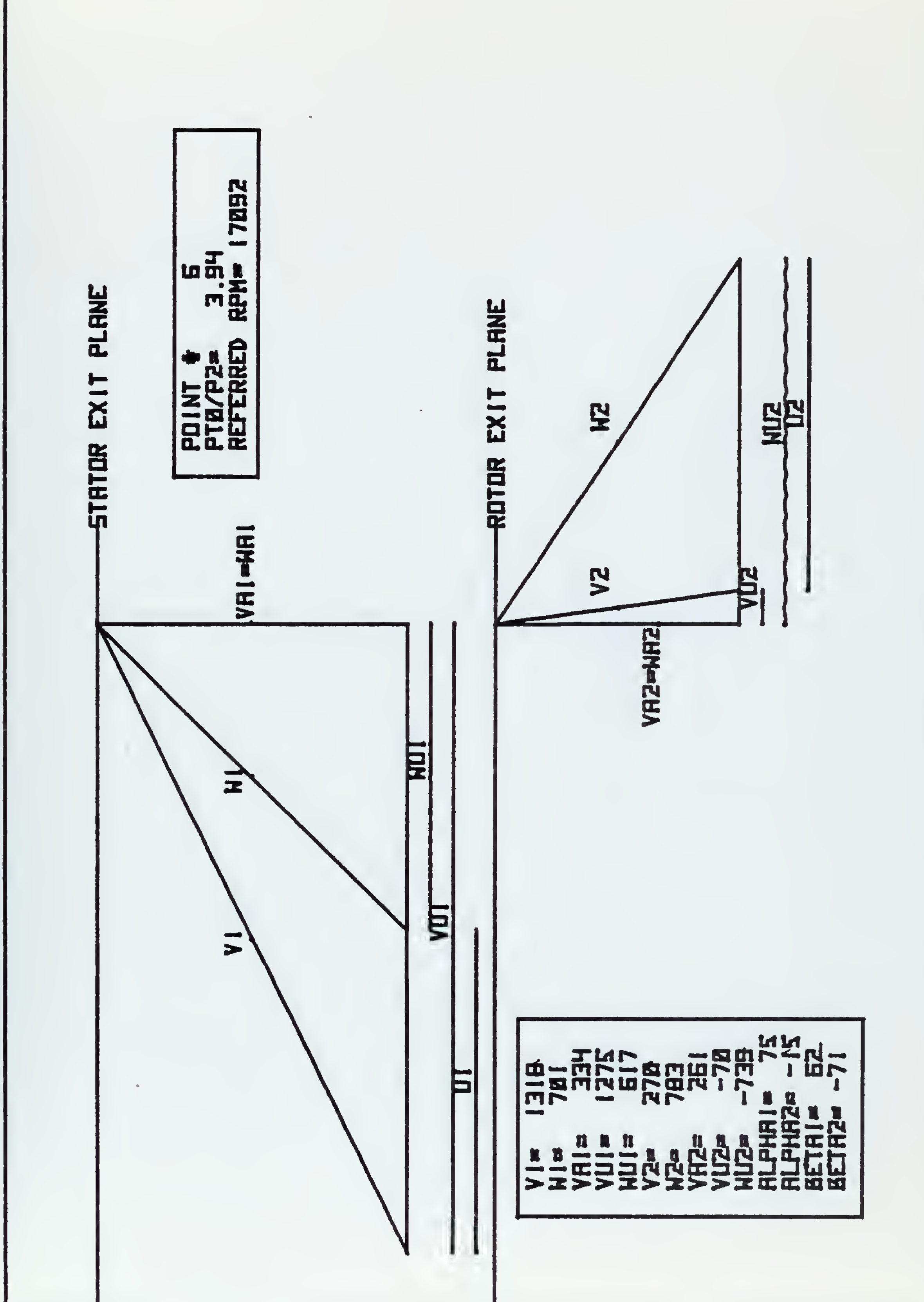


FIGURE 28 VELOCITY DIAGRAM FOR TURBINE TEST

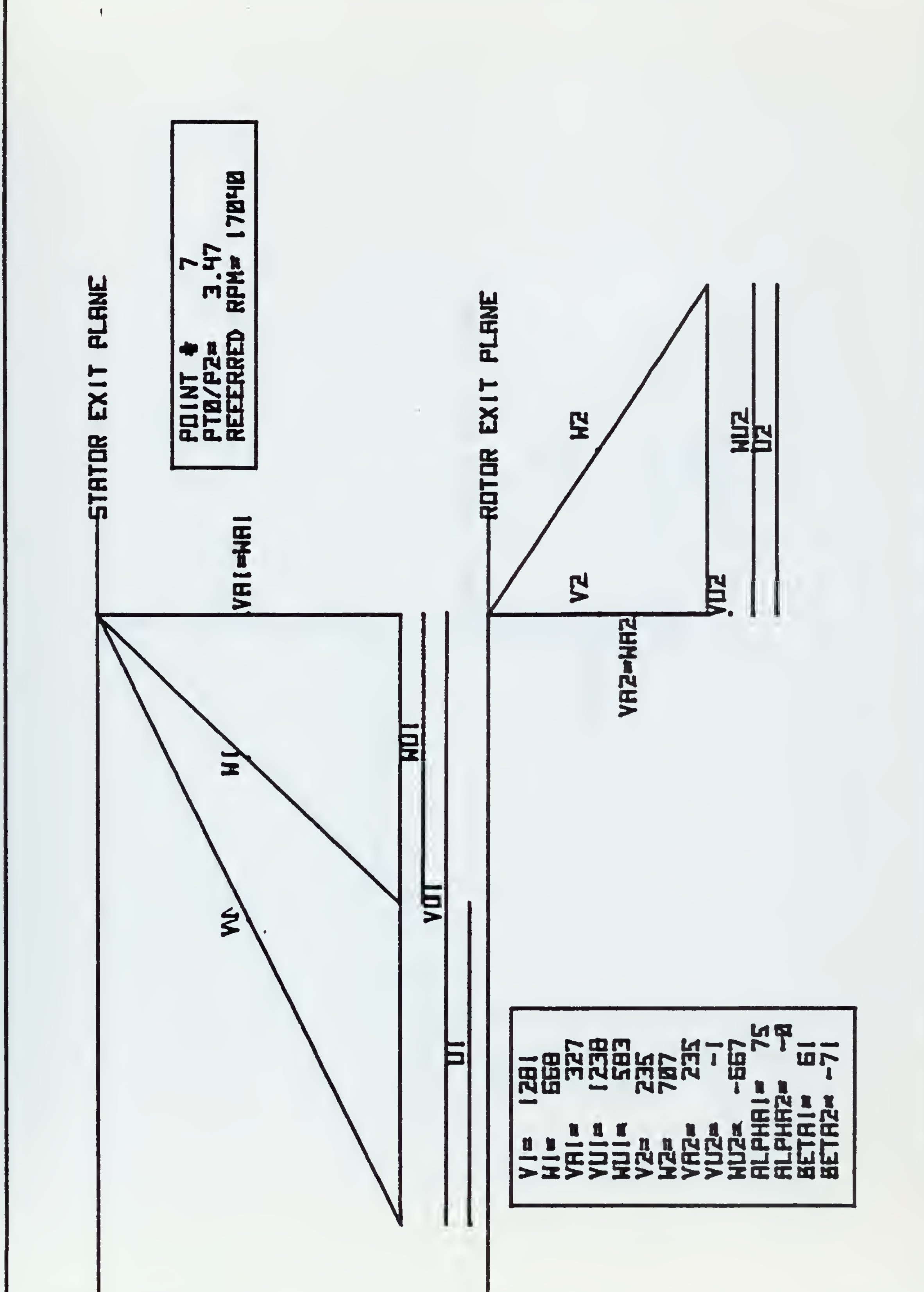


FIGURE 29 VELOCITY DIAGRAM FOR TURBINE TEST

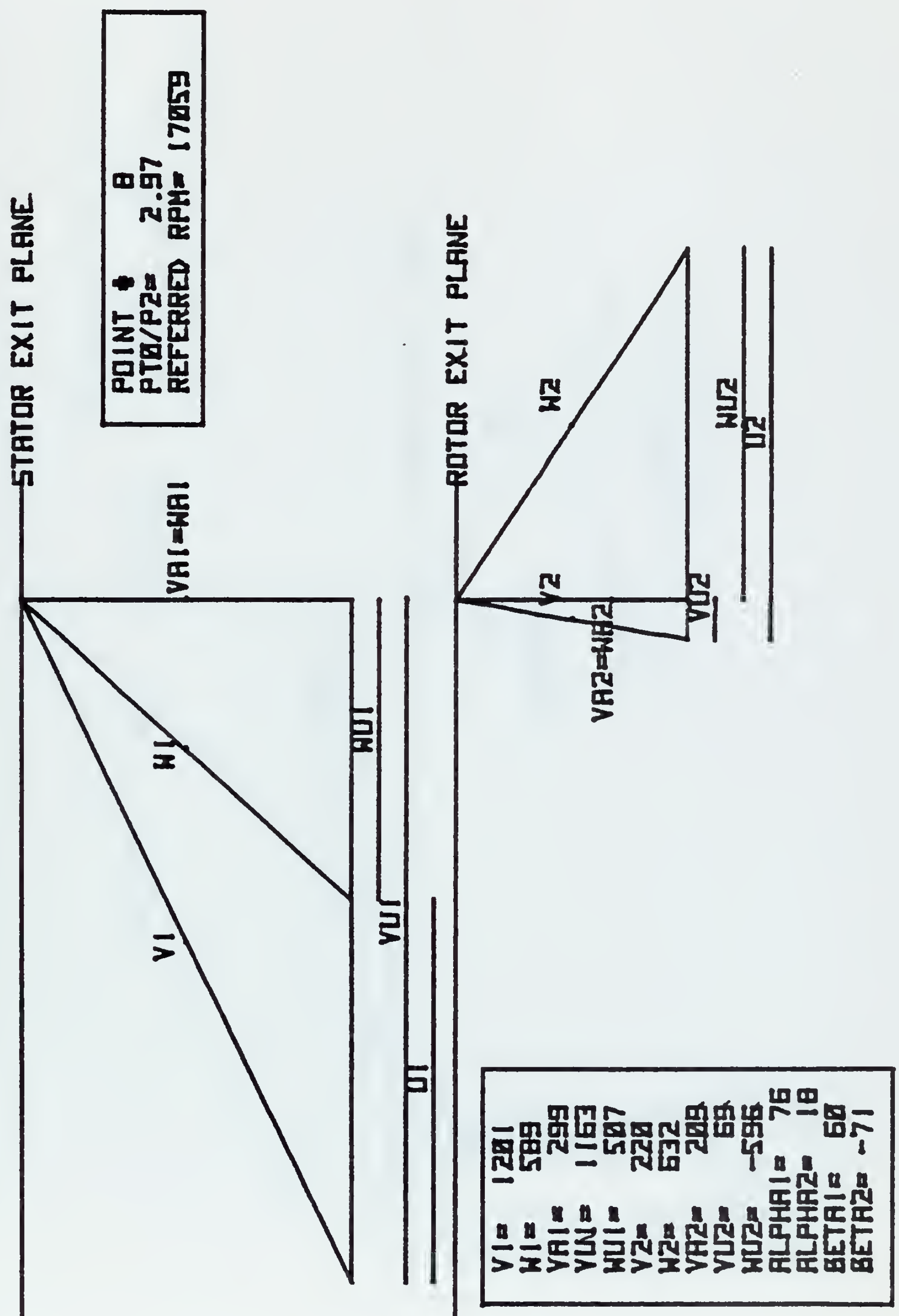


FIGURE 30 VELOCITY DIAGRAM FOR TURBINE TEST

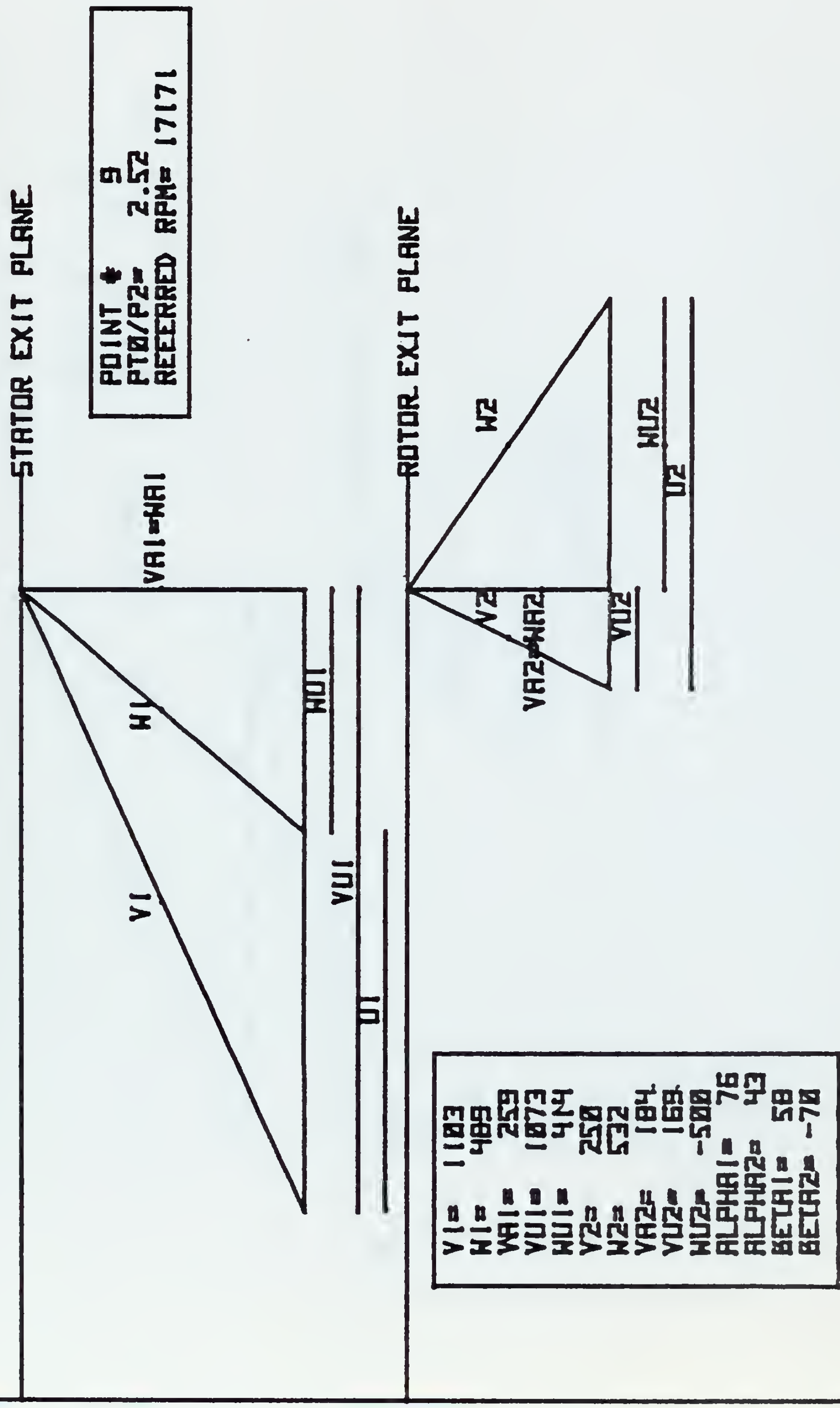
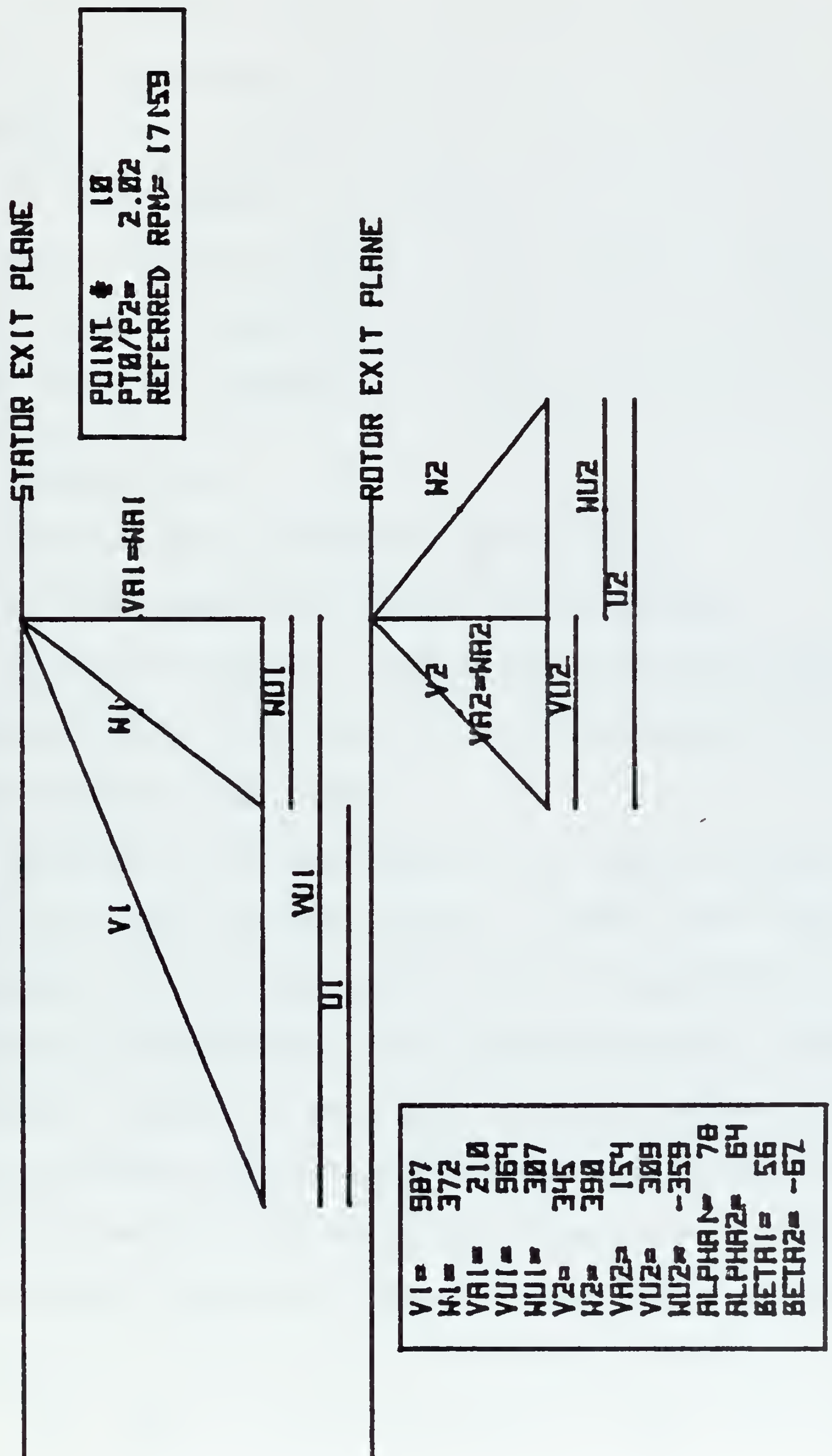


FIGURE 31 VELOCITY DIAGRAM FOR TURBINE TEST



APPENDIX A

TURBINE PERFORMANCE ANALYSIS

A-1 INTRODUCTION

This appendix gives a detailed description of the theory and analytical technique used to predict the performance of a turbine in the Turbine Test Rig. Variables used in this appendix are defined as they are introduced. Also given are the details of the computer program used to implement the analytical method. The variables used in the computer program are listed and defined in Table A-I.

A-2 ASSUMPTIONS

The following assumptions were made:

1. The stage was choked in the stator.

2. In solving the continuity equation between the stator and rotor the solution corresponding to the lower static pressure was taken.

Assumption (1) was made on the basis of past experience with the test rig. Ref. 1 states that there is supersonic flow in the stator and an examination of the pressure distribution in the stator passages confirms this statement. Fig. A-1 is a plot of the pressure distribution through the stator by pressure tap number (See Fig. 6). It can be seen in this plot that there was a sharp pressure rise between tap #3 and tap #4, which indicates the presence of a shock in the divergent section of the passage. Also, the level of static pressure at the throat taps

in comparison to the supply pressure, was observed to be independent of downstream pressure for all operating conditions.

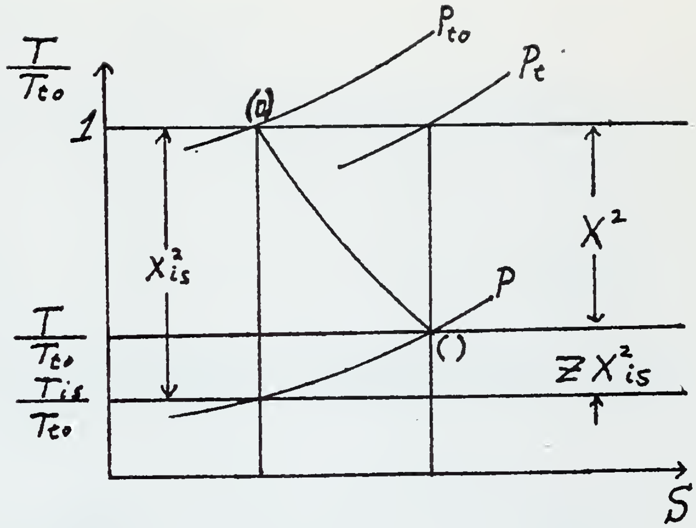
Assumption (2) limits the range of possible solutions as shown in Fig. A-2. This figure is a plot of flow coefficient ($\emptyset$) vs. pressure ratio (P_1/P_{to}) for various loss coefficients (z). (The flow coefficient, a non-dimensional mass flux, is introduced in Section A-2.) There are two possible pressures for a given flow coefficient and given loss coefficient. Solutions were limited to the left side of the line connecting the points of maximum flow function. The procedure was chosen because it resulted in the "supersonic root" for the flow from the stator. Also, conditions corresponding to choking were sought for the rotor, and the locus of locally sonic conditions is as shown in Fig. A-3. It should be noted that because of the definitions of the flow coefficient and loss coefficient, the "choking" condition at which the downstream pressure has no effect upstream of the plane in question, does not correspond to the maximum mass flux from given stagnation conditions.

The analysis is pseudo-1-dimensional. Blockage factors are introduced with the physical cross sectional area to account for non-uniform flow conditions. Loss coefficients are defined on the basis of kinetic energies. The analysis, which requires only mass flow continuity through the stage, is divided into three stages; the stator, the interblade

region and the rotor.

While the analysis itself is general, solutions can only be obtained by assigning values to chosen parameters. In the solutions reported here, the flow angle at the stator exit (α_1), the blockage factors for the stator throat (k^*_{bS}) and rotor exit plane (k_{bR}), and the rotor passage loss coefficient (z_R) were given values. The value of α_1 was fixed at $\alpha_1 = 75^o$ based on the results of the turbine tests and from the blading geometry. The blockage factors were set at $k^*_{bS}=.965$ and $k_{bR}=.85$. The rotor passage loss coefficient (z_R) was an input variable which was set at .2514 (theoretical value) or .1 (lowest value obtained in test results). The "rotor loss coefficient" output from the program, z_{1-2}, includes all losses from the stator exit plane to the rotor exit plane. z_{1-2} corresponds to the "rotor loss coefficient" evaluated from test rig measurements: consequently, $z_{1-2} \gg z_R$.

A-3 BASIC RELATIONS



The sketch shows a general adiabatic process with entropy increase (losses) on a T-S diagram on which the temperature scale has been divided by the constant stagnation temperature of the process. A perfect gas is assumed. (0) represents the initial stagnation state and ( ) represents the final state.

The non-dimensional velocity, X, is defined as $X = \frac{V}{V_{to}}$, where $V_{to} = \sqrt{2C_p T_{to}}$ is the "total" or "limiting" velocity, and the loss coefficient, z, is defined as shown. Note that the stagnation velocity is constant throughout the process.

The "flow coefficient", ϕ, is defined here as

$$\phi = \frac{\dot{w}}{\rho_{to} V_{to} A k_b} \qquad A(1)$$

where $\dot{w}$ is the flow rate, ρ_{to} is the density at the initial stagnation temperature and pressure, A is the flow area and k_b is the blockage factor. The flow coefficient defined in this way is a non-dimensional flow rate per unit area, or mass flux, referred to initial stagnation conditions.

The flow rate is given by

$$\dot{w} = \rho A k_b V$$

so that Eq. A(1) can be written

$$\phi = \frac{\rho V}{\rho_{to} V_{to}} = \left(\frac{P}{P_{to}}\right)\left(\frac{T_{to}}{T}\right)\left(\frac{V}{V_{to}}\right)$$

which, in terms of the non-dimensional velocity defined above, becomes

$$\phi = \left(\frac{P}{P_{to}}\right)\left(\frac{X}{1 - X^2}\right) \qquad \text{A(2)}$$

The loss coefficient is defined, as shown in the above sketch, as

$$z = \frac{T - T_{is}}{T_{to} - T_{is}} = 1 - \frac{X^2}{X_{is}^2} \qquad \text{A(3)}$$

Rearranging Eq. A(3),

$$(1 - z) = \frac{X^2}{X_{is}^2} = \frac{X^2}{1 - \frac{T_{is}}{T_{to}}}$$

and using the isentropic relationship between pressure and temperatures

$$X = \sqrt{(1-z)\left[1 - \left(\frac{P}{P_{to}}\right)^{\frac{\gamma-1}{\gamma}}\right]} \qquad \text{A(4)}$$

Using Eq. A(4), Eq. A(2) becomes a single equation for ϕ as a function of pressure ratio P/P_{to}. In analysing the turbine, generally the flow rate is known so that the flow coefficient can be calculated using Eq. A(1). On specifying a value for the loss coefficient, (z), the pressure ratio can be calculated from Eq. A(2) using Eq. A(4). An iterative technique is used (Newton's Method) starting with an initial estimate of the pressure ratio. Fig. A(2) shows ϕ as a function of P/P_{to} for various values of z,

in graphical form. The solution to the left of the maxima is obtained by beginning the iteration with a small value of P_1/P_{to}.

The analysis of the turbine begins by specifying that the flow rate is set by the choking of the stator nozzles. The pressures at successive stations are then calculated in turn from the flow coefficient as described above.

A-4 STATOR EXIT CONDITIONS

The flow coefficient at the throat of the stator is the value of the flow coefficient corresponding to locally sonic conditions at the minimum area. In the present work, the maximum flow coefficient at a loss coefficient equal to .05 (defined as ϕ^*) was taken at the stator throat, throughout the calculations.

Since

$$\phi^* = \frac{\dot{w}}{\rho_{to} V_{to} A_s k_{bs}} \qquad A(5)$$

where A_s and k_{bs} are the area and blockage factor at the throat station, the flow rate being constant through the machine requires that

$$\dot{w} = \rho_{to} V_{to} A_s k_{bs} \phi^* = \text{constant}$$

at stations ahead of the rotor, where T_{to} is constant; this implies that

$$P_{to} A_s k_{bs} \phi^* = \text{constant}$$

The value of ϕ^* can be obtained analytically when the loss coefficient is known. The maximum occurs where $\frac{d\phi}{d(\frac{P}{P_{to}})} = 0$, which leads to the solution (denoting values at the stator throat by an asterisk),

$$(X^*)^2 = B - \sqrt{B^2 - C} \qquad A(6)$$

where $B = \frac{\gamma + \left(\frac{\gamma-1}{2}\right)z}{\gamma+1}$, $C = (1-z)\left(\frac{\gamma-1}{\gamma+1}\right)$

and $$P^*/P_{to} = \left(\frac{D}{1+D}\right)^{\frac{\gamma}{\gamma-1}} \qquad A(7)$$

where $D = \left(\frac{2\gamma}{\gamma-1}\right)\left(\frac{1-X^{*2}}{1+X^{*2}}\right)$

Then ϕ^* is given by Eq. A(2).

With ϕ^* known, continuity of flow rate requires that at the stator exit,

$$\phi_1 = \phi^* \frac{A_s k_{bs}}{A_1 k_{b1} \cos\alpha_1} \qquad A(8)$$

where subcript 1 represents the stator exit plane (station 1) and α_1 is the flow angle at thestator exit.

Knowing ϕ_1, P_1/P_{to} can be found by iteration as described in section A-3. Solutions on the left side of Fig. A-2 were selected by beginning the iteration with P_1/P_{to} close to zero.

After obtaining thevalue for P_1/P_{to} then X_1 can be

calculated using

$$X_1 = \sqrt{(1-z_s)\left[1-\left(\frac{P_1}{P_{to}}\right)^{\frac{\gamma-1}{\gamma}}\right]} \qquad A(9)$$

Then, P_1/P_{t1} can be found from

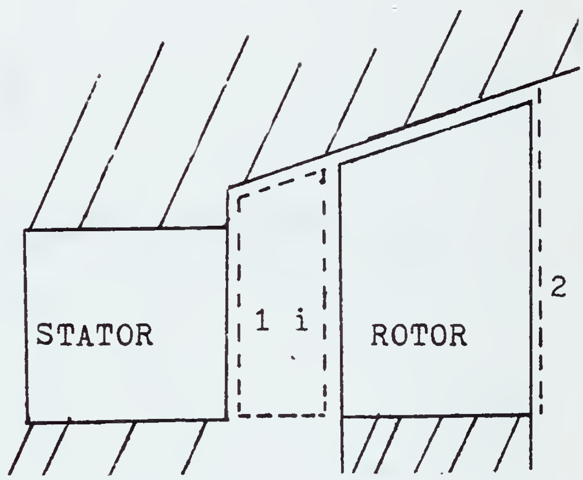
$$P_1/P_{t1} = (1-X_1^2)^{\frac{\gamma}{\gamma-1}} \qquad A(10)$$

and

$$P_{t1}/P_{to} = (P_1/P_{to})\,(P_{t1}/P_1) \qquad A(11)$$

A-5 INTERBLADE SPACE

The interblade space, from Station (1) to Station (i), is shown in the sketch.



Conditions at (i) are calculated in a manner similar to those at Station (1).

First, by continuity,

$$\phi_i = \phi^* \frac{A_s\, k_{bs}}{A_i\, k_{bi} \cos\alpha_i}\;\frac{P_{to}}{P_{t1}} \qquad A(12)$$

where α_i is the flow angle at the rotor entrance.

The value of α_i is obtained from the condition that angular momentum is conserved in the interblade space. Then

$$X_{ui} = \frac{R_1}{R_i} X_1 \sin\alpha_1 \qquad A(13)$$

where R_1=mean radius at Station 1, and R_i=mean radius at Station i.

Then

$$\alpha_i = \sin^{-1} \frac{X_{ui}}{X_i} \qquad A(14)$$

and, also

$$\bar{U}_i = \left(\frac{\pi N}{360} \cdot R_i\right) \Big/ V_{to} \qquad A(15)$$

where N is the RPM, and the tangential velocity (V_{ui}) has been non-dimensionalized as

$$X_{ui} = \frac{V_{ui}}{V_{to}} \qquad A(16)$$

Knowing ϕ_i, a value for P_i/P_{t1} can be found if a value of the interblade loss coefficient (z_i) is assumed. Then

$$P_i/P_{to} = (P_i/P_{t1})\,(P_{t1}/P_{to}) \qquad A(17)$$

and

$$X_i = \sqrt{(1-z_i)\left[1-\left(\frac{p_i}{p_{t1}}\right)^{\frac{\gamma-1}{\gamma}}\right]} \qquad A(18)$$

With the absolute flow properties established at the rotor entrance, the relative flow conditions can be calculated. First, from the geometry of the velocity diagram, the relative flow angle (β_i) is given by

$$\beta_i = \tan^{-1}\left[\frac{X_i \sin\alpha_i - \overline{U}_i}{X_i \cos\alpha_i}\right] \qquad A(19)$$

where

$$\overline{U}_i = \frac{U_i}{V_{to}} \qquad A(20)$$

is the non-dimensional rotor speed at radius R_i. Then the non-dimensional relative velocity (X_{wi}) is, from the velocity diagram, given by

$$X_{wi} = \frac{\cos\alpha_i}{\cos\beta_i} \qquad A(21)$$

and the temperature T_i, by

$$\frac{T_i}{T_{to}} = 1-X_i^2 \qquad A(22)$$

The equivalent temperature (Ref. 1), is given by

$$\frac{T_{Ei}}{T_{to}} = 1 - X_i^2 + X_{wi}^2 + \overline{U}_i^2\left[\left(\frac{R_2}{R_i}\right)^2 - 1\right] \qquad A(23)$$

and therefore

$$\frac{P_{Ei}}{P_{to}} = \frac{P_{Ei}}{P_i}\frac{P_i}{P_{to}} = \left(\frac{T_{Ei}}{T_{to}}\frac{T_{to}}{T_i}\right)^{\frac{\gamma}{\gamma-1}}\frac{P_i}{P_{to}} \qquad A(24)$$

where

T_{Ei} = equivalent temperature into rotor (See Fig. A-7)

P_{Ei} = equivalent pressure into the rotor (See Fig. A-7)

A-6 ROTOR CONDITIONS

Using the values calculated thus far it is now possible to calculate the conditions in the rotor. Continuity requires that

$$\phi_R = \phi^* \frac{\sqrt{\frac{T_{Ei}}{T_{to}}}}{\left(\frac{P_{Ei}}{P_{to}}\right)}\left(\frac{A_s}{A_R}\frac{k_{bs}}{k_{bR}}\right) \qquad A(25)$$

where a subscript R denotes the exit plane in the rotor blading.

It should now be noted that the process in the rotor frame from the equivalent conditions at (i) to the exit of the rotor is entirely similar to the basic process described in Section A-3. However, the equivalent temperature takes the place of stagnation temperature and relative velocity replaces velocity in the given equations.

The pressure ratio which satisfies the flow coefficient given by Eq. A(25), for an assumed rotor passage loss coefficient (z_R) is P_2/P_{Ei}. This is obtained as described in Section A-3. The corresponding non-dimensional velocity is now

$$Y_{w2} = \sqrt{(1-z_R)\left[1-\left(\frac{P_2}{P_{Ei}}\right)^{\frac{\gamma-1}{\gamma}}\right]} \qquad A(26)$$

where

$$Y_{w2} = \frac{W_2}{\sqrt{2C_p T_{Ei}}} \; . \qquad A(27)$$

Hence

$$X_{w2} = Y_{w2}\sqrt{\frac{T_{Ei}}{T_{to}}} \; . \qquad A(28)$$

From the velocity diagram,

$$X_2^2 = (\overline{U}_2 - X_{w2}\sin\beta_2)^2 + (X_{w2}\cos\beta_2)^2 \qquad A(29)$$

where $\beta_2 = 71^{\circ}$ (from the geometry of the rotor), and $\overline{U}_2 = \overline{U}_i \dfrac{R_2}{R_i}$.

The temperature at the rotor exit is given by

$$\frac{T_2}{T_{to}} = \left[(1-z_R)\left(\frac{P_2}{P_{Ei}}\right)^{\frac{\gamma-1}{\gamma}} + z_R\right]\left(\frac{T_{Ei}}{T_{to}}\right) \qquad A(30)$$

so that $$\frac{T_{t2}}{T_{to}} = \frac{T_2}{T_{to}} + X_2^2 \qquad A(31)$$

Next the loss coefficient that includes all of the losses in the stage from Station 1 to Station 2 can be calculated. This coefficient includes all of the losses in the inter-blade region as well as those in the rotor passage. It is called z_{1-2} here but, in fact corresponds to the measured rotor loss coefficient in the TTR since all losses aft of the stator in the TTR are included in the "rotor loss coefficient."

$$z_{1-2} = 1 - \frac{T_E - T_2}{T_E - T_{2_{is}}}$$

$$= 1 - \frac{\dfrac{T_E}{T_{to}} - \dfrac{T_2}{T_{to}}}{\dfrac{T_E}{T_{to}} - \dfrac{T_{2is}}{T_{to}}}$$

$$= 1 - \frac{\dfrac{T_E}{T_{to}} \quad \dfrac{T_2}{T_{to}}}{\dfrac{T_E}{T_{to}} \quad \left(\dfrac{P_2}{P_{to}} \dfrac{P_{to}}{P_{t1}}\right)^{\frac{\gamma-1}{\gamma}}} \qquad A(32)$$

A-7 STAGE PERFORMANCE PARAMETERS

In the notation used for the turbine test rig in Ref. 1 and Ref. 2, the following equations determine the turbine performance parameters from quantities calculated in the above steps:

$$\Delta T_w = T_{to}\left(1 - \frac{T_{t2}}{T_{to}}\right) \qquad A(33)$$

$$\Delta T_{is} = T_{to}\left[1 - \left(\frac{P_2}{P_{to}}\right)^{\frac{\gamma-1}{\gamma}}\right] \qquad A(34)$$

$$\eta_{T-S} = \frac{\Delta T_w}{\Delta T_{is}} \qquad A(35)$$

$$\dot{w} = \emptyset^* \, (\rho_{to} V_{to} A_o k_{bo}) \qquad A(36)$$

$$H.P. = \dot{w} \, (.2402) \, \frac{778}{550} \, \Delta T_w \qquad A(37)$$

$$M = H.P. \, (550) \, \frac{360}{\pi N} \qquad A(38)$$

$$\delta = \frac{P_{to}}{P_{ref}} \qquad A(39)$$

$$\theta = \frac{T_{to}}{T_{ref}} \qquad A(40)$$

$$\dot{w}^* = \frac{\dot{w}\sqrt{\theta}}{\delta} \qquad A(41)$$

$$H.P.^* = \frac{H.P.}{\delta\sqrt{\theta}} \qquad A(42)$$

$$M^* = \frac{M}{\delta} \qquad A(43)$$

$$N^* = \frac{N}{\sqrt{\theta}} \qquad A(44)$$

A-8 THE COMPUTER PROGRAM

A-8.1 Overview

There are nine degrees of freedom in the computer program:

1. α_1, the flow angle out of the stator
2. k_{b1}, the stator exit blockage factor
3. k_{bi}, the interstage blockage factor
4. k_{bR}, the rotor passage blockage factor
5. z_s, the stator loss coefficient
6. z_i, the interstage loss coefficient
7. z_R, the rotor passage loss coefficient
8. N, turbine RPM
9. T_{to}, total temperature upstream of the stator

α_1 is input on the basis of turbine test results. At high pressure ratios the value of α_1 was measured to be 75^o-76^o. Therefore, the value of α_1 was set and held at 75^o. The values of RPM and T_{to} are those conditions for which a solution is being sought.

All the other parameters, the rotor passage loss coefficient (z_R) and blockage factor (k_{bR}) were given chosen values. This choice was made because the incidence losses were included in the interblade calculation. It was thought that the parameters describing the flow inside the rotor passages could be held fixed as the speed was allowed to vary.

Consequently, there are four degrees of freedom with which the program works. The goal for the program is to

find those solutions which result in flow coefficients in the rotor that are less than, or equal to, the "choking" value. This is done in the program by iteration on the remaining four degrees of freedom. Fig. A-5 is a block diagram showing schematically the iteration process. The order of iteration is (from the most frequent to the least frequent), z_i, z_s, k_{bi}, and k_{b1}. The loss coefficients for both the stator and interblade always begin at zero and they are incremented as the program seeks solutions. A graphical representation of a typical iteration is shown in Fig. A-4. It can be seen that the program can find a solution for the condition where the loss coefficient is equal to zero (denoted as (1) on Fig. A-4). On the next iteration the program will set the loss coefficient equal to .1 and it can be seen that a solution still exists at point (2). On the next iteration, however, it can be seen that a solution does not exist for a loss coefficient equal to .2. In this case the program will go back and add a smaller increment to the previous loss coefficient (.1).

Each time an added increment results in a loss coefficient greater than the maximum allowable the program subtracts that increment and adds a smaller one. In this way the program eventually finds the maximum loss coefficient which will yield a solution. At this point, the given flow coefficient is at the maximum point for the calculated loss coefficient line. At this point "choking", as defined here, has occurred.

It should be pointed out that this "choking" condition (the point of maximum flow coefficient for any given loss coefficient) is not the point corresponding to locally sonic conditions. In Fig. A-3 the locus of points of maximum flow coefficient and the locus of points of local sonic flow are both shown. The point of locally sonic flow always falls to the left of the "choking" point (as defined in this work), and the flow coefficient for sonic flow is always less than the flow coefficient at the defined choking point. The reason is that the point of maximum mass flux (maximum flow coefficient) is for given upstream stagnation conditions (see next section). Because of the way the flow coefficient is defined, the maximum for a given loss coefficient will not correspond to local sonic conditions except where the loss coefficient is equal to zero.

The difference between the "choking" flow coefficient corresponding to the maximum and the flow coefficient at the local sonic condition is about 10% at a loss coefficient of .3 (the highest rotor loss coefficient predicted by the program). The difference is about 4% at a loss coefficient equal to .25 (the highest stator loss predicted), and insignificant (less than 1%) at a loss coefficient of about .15.

A-8.2 Description of the Program

The following discussion refers to Fig. A-6, which is a detailed flow chart of the program, to Table A-I, which is a list of program variables and their definitions, and to the program listing given in Section A-10.

The first step in the program is to input the variables. α_1 is input in line 280 (it is also converted to radians in that line). k_{b1} is input in line 286 and k_{bi} is input in line 284. All other variables are input as requested by the program (lines 20-110). Note that P_2 and P_{to} are input also. They are not involved in the calculations and were originally input as the conditions for which a particular solution was sought.

The next step is to go to a subroutine to calculate the value of ϕ^* (line 170). ϕ^* is defined as the value of the flow coefficient at the throat of the stator which sets the values of the flow coefficients at all downstream stations. ϕ^* is generated in the subroutine by the method covered in Section A-4 assuming a loss coefficient of .05 to the throat of the stator (line 160).

Next the program calculates the conditions in the stator exit plane. This is done by first calculating the value for ϕ_1 (line 320). As explained earlier, the value for ϕ_1 determines the upper limit of the stator loss coefficient.

Using the method described in Section A-3, the program calculates the value for P_1/P_{to}. Note that if the stator

loss coefficient is large enough, so that a solution does not exist, this will be detected in the decision, "$P_1/P_{to} < 0$ or > 1" (lines 350 and 370). If "yes" then the stator loss (z_s) is too large and the following steps are taken:

1. The last increment added to the stator loss is subtracted (line 420). (Note, the initial stator loss increment is .1.)

2. The existing stator loss increment is multiplied by some factor less than 1 (line 430).

3. The new, smaller, increment is then added back to the stator loss coefficient (line 1570).

If stator loss increment is sufficiently small (line 1575) then the upper limit of the loss coefficient has been reached as discussed earlier. At this point no further solutions are attainable and k_{bi} is incremented (line 2290). Note that the upper limit of stator loss will not normally be attained until several iterations have been made through the entire program.

Next the interblade calculations are made. The solution for P_1/P_{to} found for the stator exit plane is used to make the calculations for ϕ_i (line 770). Again, ϕ_i determines the upper limit on the interblade loss. In the interblade region, however, there is another criterion that must be met. The value for β_i generated within the program must be equal to β_8 ($\beta_8 = 69^{\circ}$ and is fixed by the geometry). This criterion is met by iterating the interblade loss coefficient as follows: On every pass through

this section the interblade loss is initially set to zero and the interblade loss increment is initially set to .1 (lines 720, 740). The value for P_i/P_{t1} is calculated using the same procedure as described earlier for P_1/P_{to}. Again, the value of P_i/P_{t1} is checked on each iteration to ensure that it falls between 0 and 1 (line 780). If it does not then the interblade loss is reduced in a method analagous to that covered for the stator loss.

After generating a value for P_i/P_{t1} the program calculates β_i (line 1208). When the value for β_i is generated it is compared to β_B and three results are possible. If β_i is equal to $\beta_B \pm .05$ then convergence has occurred and the program will continue. If β_i is less than β_B (line 1250) then the value for interblade loss coefficient is increased by the current value of interblade increment and the program goes back to the start of the interblade calculations and begins anew. If β_i is greater than β_B (line 1240) the value of interblade loss is decreased in the same way as was described for the stator loss.

Two results are possible in the interblade region: Either the program finds the value of the interblade loss that gives flow angle convergence ($\beta_i = \beta_B$), or the interblade loss increment becomes sufficiently small (10^{-6}) to trigger termination.

Note that the interblade loss increment is allowed to become very small (10^{-6}) (line 850). This is because the above mentioned angle convergence requirement is very

sensitive. Note, also, that when the interblade loss increment falls to 10^{-6} and the iteration process in the interblade region is terminated, the stator loss coefficient is incremented (line 850). Stator loss is incremented because a higher value of stator loss may result in conditions that allow convergence in the interblade region. If not, the stator loss calculations eventually reach the termination criterion mentioned earlier.

Once angle convergence has been attained in the interblade region then the value of the rotor flow coefficient can be calculated (line 1470). If the program gets to this point for any given set of conditions then a solution exists and the only determination to be made is whether the resulting rotor flow coefficient is greater than the "choking" value (line 1530). If "yes" then it is disallowed, since a flow coefficient greater than the "choking" value cannot occur physically. If the calculated coefficient is less than, or equal to the "choking" value then the results are printed, the stator loss is incremented, and the calculations start again at the stator exit plane.

The choking value of the flow coefficient for the rotor (ϕ^*_R) is determined in exactly the same way as the choking value for the stator (ϕ^*), with the assumed value of rotor loss used as the loss coefficient. (At present the value for ϕ^*_R is calculated separately and inserted in line 121. It is suggested that a "GOSUB 2140" be inserted at this point. Then the input value for rotor loss

coefficient can be changed easily and the "choking" value for the rotor will be calculated in the program.) If the resulting flow coefficient is greater than "choking", the program increments stator loss coefficient and returns to the stator plane calculations to begin anew.

Therefore, in all cases the program will seek the highest value of stator loss coefficient for which solutions exist. At some point, the rotor flow coefficient normally attains a value less than the "choking" value and all solutions for higher stator loss coefficients will be acceptable solutions. Or, if the "choking" condition is never reached, the program will attain the highest possible stator loss in attempting to achieve solutions less than "choking".

When the highest value of the stator loss is reached, there are two possible avenues in the program. If the program was generating solutions less than "choking" at the time of termination in the stator calculations, then k_{bi} will be incremented, stator loss will be set back to zero, and the process begun anew (line 2280). If the solutions being generated at the time of termination were greater than "choking", then the upper value of k_{bi} has been reached and any increase in k_{bi} will yield no further solutions. In this case k_{b1} is incremented, k_{bi} is set to the pre-determined lower limit, and stator loss is reset to zero (line 2330).

The program continues in the above manner until the

pre-determined upper limit of k_{b1} is reached (line 2335), at which point the program is terminated.

Since the 21-MX computer has only 6 digits of accuracy, round-off error has been a problem. Where Newton's Method is used to solve for pressure ratio the computer lacks sufficient accuracy to converge to the desired accuracy under certain conditions. When this happens, the program will stay inside the iteration and never converge. This problem has been solved by putting a counter inside each iteration loop (lines 540 and 980). More than 60 iterations is treated as a non-convergent condition, and the appropriate loss coefficient is reduced in the manner covered previously.

A-9 OPERATING PROCEDURE

The procedures for operating the Hewlett-Packard 21-MX computer will not be covered in this paper since the applicable manuals are available at the laboratory. The computer must be on, with the RTE-B operating system "READY".

1. Load the paper tape program labeled "TTR 11".

2. Edit into the program the minimum values of k_{b1} (line 284) and k_{b1} line (286). Edit the desired value of α, into line 280. If it is desired to get a print-out of every solution then either remove lines 1724 and 1726 or place "REM" in front of them. If these two lines are left in the program then a solution will be printed out only if the calculated pressure ratio (P_2/P_{to}) is less

than, or greater than, the previously calculated lowest or highest pressure ratio respectively, for the value of k_{b1} then being used.

3. Type "RUN", "RETURN" on the keyboard and the program will begin execution by asking for inputs:

"INPUT STATOR LOSS COEFFICIENT" The normal input is zero. However, if some specific case is required then any desired loss coefficient can be input.

"INPUT PTO" Input the upstream total pressure.

"INPUT P2" Input the hood pressure.

"INPUT RPM" Input the RPM desired.

"INPUT TTO" Input the total temperature into the stage.

This is all that is required to operate the program. Depending on the range of k_{b1} requested the program will take from 30 minutes to 36 hours to run on the 21-MX.

It is important to realize that the above procedure will produce solutions that give values of the rotor flow coefficient less than "choking". If the turbine performance at the "choking" condition is desired then after the computer run is complete a further step must be taken. It is necessary to take the values of k_{b1} and k_{bi} for which a solution is desired and force the choking condition. This is done as follows:

1. Scan the output results at the desired k_{b1} and k_{bi} and locate the point that has a value of rotor flow coefficient closest to the choking value and a non-zero

stator loss coefficient.

2. Re-start the program, this time putting in a value of the stator loss coefficient slightly less than the value printed on the output.

As stator loss is manually decreased, the rotor flow coefficient will increase towards the choking value. When the rotor flow coefficient is within the limits specified in line 2370 the words "CONVERGENT CONDITIONS" will appear on the teletype output preceding the printed results.

If the stator loss coefficient is decreased too much, the rotor flow coefficient will increase to a value above choking. In this case no ouput will be printed on the teletype. However, the flow coefficient will appear on the video display. In this case increase the stator loss slightly.

It has been found in this work that the extreme values of referred horsepower, stator loss, rotor loss, and efficiency occur at the lowest and highest values of k_{bi} for each value of k_{b1}. With this in mind it is possible to find the range of predicted values at choking by forcing to choking only the lowest value of k_{bi} (with non-zero stator loss) and the highest value of k_{bi} for each k_{b1}.

A-10 PROGRAM LISTING

```
10  REM***TTR1(VI)**THIS PROGRAM IS TO PREDICT THE LOSSES THRU THE TTR
11  LET Q8=0
12  LET Q9=1
15  LET C1=1
20  PRINT# 1"INPUT STATOR LOSS COEFFICIENT"
30  INPUT Z0
40  PRINT# 1"INPUT PT0"
50  INPUT P0
60  PRINT# 1"INPUT P2"
70  INPUT P9
80  PRINT# 1"INPUT RPM"
90  INPUT N
100  PRINT# 1"INPUT TT0"
110  INPUT T0
121  LET F4=.214185
122  LET R8=.3
125  REM G0=(GAMMA-1)GAMMA
130  LET G0=(1.402-1)/1.402
150  LET P2=P9/P0
155  REM L1= LOSSES IN STATOR, UP TO THROAT, FOR CALCULATING PHI(*)
160  LET L1=.05
165  REM GOSUB TO CALCULATE PHI(*)
170  GOSUB 2140
180  LET X8=SQR(.95*(1-P8↑G0))
190  LET F0=P8*(X8/(1-X8↑2))
200  PRINT# 1"PHI(*)="F0
225  REM CALCULATE RHO(TOTAL)
230  LET R0=(P0/29.92)*(492/T0)*.080722
235  REM CALCULATE V(TOTAL)
240  LET V0=109.62*SQR(T0)
245  REM INPUT AREA OF STATOR THROAT
250  LET A0=.020625
255  REM INPUT BLOCKAGE FACTOR FOR STATOR
260  LET K1=.965
265  REM INPUT AREA OF INTERSTAGE REGION
270  LET A1=.105423
```

```
275 REM INPUT INITIAL GUESS FOR ALPHA1
277 LET J=3.14159
280 LET O1=75*J/180
281 REM INPUT BLOCKAGE FOR STATOR
282 LET K5=.851
283 REM INPUT INTERSTAGE BLOCKAGE
284 LET K4=.58
285 REM INPUT "1" BLOCKAGE
286 LET K6=.772
291 REM Z8=INITIAL STATOR LOSS INCREMENT
292 LET Z8=.1
300 LET P1=.001
310 REM CALCULATE PHI1
320 LET F1=F0*(A0*K1)/(A1*K6*COS(O1))
330 LET Y9=0
340 REM THE FOLLOWING CALCULATES P1/PT0
350 IF (P1>0) THEN 370
360 GOTO 380
370 IF ((1-Z0)*(1-P1↑G0)>0) THEN 450
380 PRINT# 1"BLOWUP IN CALC. P1/PT0, EXCESS STATOR LOSS"
390 PRINT# 1"AT BLOWUP F1="F1
400 PRINT# 1"AT BLOW-UP Z0="Z0
410 PRINT# 1"DECREASE INCREMENT SIZE AND CONTINUE"
420 LET Z0=Z0-Z8
425 IF Z0<0 THEN 2280
430 LET Z8=Z8*K8
440 GOTO 1570
450 LET X1=SQR((1-Z0)*(1-P1↑G0))
460 LET A9=P1*(X1/(1-X1↑2))-F1
470 LET D1=(P1↑G0)*G0/2*((1-X1↑2)/(1-P1↑G0))
480 LET D1=(X1/(1-X1↑2))*(1-(D1*(1/(1-X1↑2)+(2*X1↑2)/(1-X1↑2)↑2)))
490 LET E=A9/D1
500 LET A8=P1-E
502 LET X1=SQR((1-Z0)*(1-P1↑G0))
504 LET W9=P1*(X1/(1-X1↑2))
```

```
510  IF F1>W9 AND (F1-W9<.000005 OR ABS(P1-A8)<.0005) THEN 635
520  LET Y9=Y9+1
530  LET P1=A8
540  IF (Y9>60) THEN 380
550  GOTO 350
635  LET P1=A8
640  LET X1=SQR((1-Z0)*(1-P1↑G0))
650  LET P3=(1-X1↑2)↑(1/G0)
660  LET P4=P1/P3
667  PRINT
668  PRINT# 1"THE FOLLOWING ARE FOR STATOR LOSS="Z0"ALPHA1="01*180/J
669  PRINT# 1"RPM="N"KI="K4"K1="K6
670  PRINT# 1"P1/PT0="P1
680  PRINT# 1"P1/PT1="P3
690  PRINT# 1"PT1/PT0="P4
700  REM CALCULATE INTERSTAGE PHI AND ITERATE WITH INCREASING
710  REM INTERSTAGE LOSS UNTIL BETA(I)=BETA(BLADE)
720  LET Z3=-.1
730  LET Z9=.1
740  LET Z3=Z3+Z9
750  LET P5=.001
760  LET Y9=0
770  LET F3=F0*(1/P4)*((A0*K1)/(K4*.134902*COS(1.20443)))
780  IF (P5<0 OR P5>1) THEN 850
790  GOTO 890
850  IF (Z9>.000001) THEN 1261
860  PRINT# 1"NO CONVERGENCE FOR THIS STATOR LOSS(Z9<.000001),"
870  PRINT# 1"DECREASE STATOR LOSS AND CONTINUE"
880  GOTO 420
890  LET X3=SQR((1-Z3)*(1-P5↑G0))
900  LET A9=P5*(X3/(1-X3↑2))-F3
910  LET D1=(P5↑G0)*G0/2*((1-X3↑2)/(1-P5↑G0))
920  LET D1=(X3/(1-X3↑2))*(1-(D1*(1/(1-X3↑2)+(2*X3↑2)/(1-X3↑2)↑2)))
930  LET E=A9/D1
940  LET A8=P5-E
```

```
942  LET X3=SQR((1-Z3)*(1-P5↑G0))
944  LET W9=P5*(X3/(1-X3↑2))
950  IF F3>W9 AND (F3-W9<.000005 OR ABS(P5-A8)<.0005) THEN 1075
960  LET P5=A8
970  LET Y9=Y9+1
980  IF (Y9>60) THEN 1000
990  GOTO 780
1000  PRINT# 1"CONVERGENCE NOT POSSIBLE FOR STATOR LOSS="Z0
1001  PRINT# 1"DECREASE STATOR LOSS AND CONTINUE"
1010  GOTO 420
1075  LET P5=A8
1080  LET P6=P5*P4
1090  LET X3=SQR((1-Z3)*(1-P5↑G0))
1100  LET Q1=(1-X3↑2)↑(1/G0)
1110  LET P=P6/Q1
1170  LET X4=(4.18375/4.195)*X1*SIN(O1)
1180  IF (X4/X3>1) THEN 1281
1190  LET O3=FNA(X4/X3)
1200  LET U3=(((J*N)/360)*4.195)/V0
1208  LET B3=(X3*SIN(O3)-U3)/(X3*COS(O3))
1210  LET B3=ATN(B3)
1215  LET B3=180*B3/J
1230  IF ((69-B3)>0 AND (69-B3)<.1) THEN 1318
1240  IF (B3>69) THEN 1290
1250  GOTO 740
1261  REM "BLOW-UP CAUSED BY EXCESSIVE INTERSTAGE LOSS"
1270  GOTO 1290
1280  PRINT
1281  REM "X4/X3>1, DECREASE INTERSTAGE LOSS AND CONTINUE"
1290  LET Z3=Z3-Z9
1292  IF (Z3<0 AND (C1=1)) THEN 2280
1293  LET C1=0
1300  LET Z9=Z9*A8
1305  IF Z3<0 THEN 2270
1308  IF Z9<.000001 THEN 860
```

```
1310 GOTO 740
1318 PRINT
1319 PRINT
1320 PRINT# 1"AT ANGLE CONVERGENCE FOLLOWING CONDITIONS EXISTED:"
1330 PRINT# 1"STATOR LOSS COEFFICIENT="Z0
1340 PRINT# 1"INTERSTAGE LOSS COEFFICIENT="Z3
1360 PRINT# 1"BETA(I)="B3
1370 PRINT# 1"P1/PT0="P1
1380 PRINT# 1"P(I)/PT1="P5
1381 LET B3=B3*J/180
1390 LET X5=X3*COS(O3)/COS(B3)
1400 REM "XW1="X5
1410 LET T3=1-X3↑2
1420 REM"T1/TT0="T3
1430 LET T4=1-X3↑2+X5↑2+U3↑2*((4.25275/4.195)↑2-1)
1440 REM "TE1/TT0="T4
1450 LET P7=(T4/T3)↑3.48756*P6
1460 REM "PE1/PT0="P7
1470 LET F2=F0*(SQR(T4)/P7)*((A0*K1)/(.04871*K5))
1475 LET O1=O1*180/J
1490 PRINT# 1"ALPHA1="O1
1491 LET O1=O1*J/180
1500 PRINT# 1"CALCULATED ROTOR FLOW COEFFICIENT="F2
1510 PRINT# 1"ISENTROPIC ROTOR FLOW COEFFICIENT (WITH LOSS=.2514)="F4
1530 IF F2>F4 THEN 1550
1540 GOTO 2370
1550 PRINT# 1"CALCULATED ROTOR FLOW COEFFICIENT>ISENTROPIC ROTOR FLOW"
1560 PRINT# 1"COEFFICIENT (WITH ROTOR LOSS=.2514).  THEREFORE, WILL"
1561 PRINT# 1"RE-COMPUTE WITH STATOR LOSS COEFFICIENT INCREMENTED"Z8
1570 LET Z0=Z0+Z8
1572 IF ((Z8<.0008) AND (F2>.22)) THEN 2330
1575 IF (Z8<.0008) THEN 2280
1580 GOTO 300
1700 REM COMPUTE P1/PT0 FOR THESE CONDITIONS AND 2D COMPARE TO THEOR.
1705 PRINT# 6"CONVERGENT CONDITIONS"
```

```
1706  LET S9=.2514
1707  GOSUB 7000
1710  LET R9=S8*P7
1720  PRINT# 1"P2/PTO(ACTUAL)="P2"      P2/PTO(CALCULATED)="R9
1721  IF (R9<Q9) THEN 1724
1722  IF (R9>Q8) THEN 1726
1723  GOTO 1570
1724  LET Q9=R9
1725  GOTO 2430
1726  LET Q8=R9
1729  GOTO 2430
2130  DEF FNA(X)=ATN(X/SQR(1-X*X))
2140  LET P8=.5
2150  LET X8=SQR((1-L1)*(1-P8↑G0))
2160  LET D1=(P8↑G0)*G0/2*((1-X8↑2)/(1-P8↑G0))
2170  LET D1=(X8/(1-X8↑2))*(1-(D1*(1/(1-X8↑2)+(2*X8↑2)/(1-X8↑2)↑2)))
2180  LET D2=((-.161604*(1-X8↑2))/(1-P8↑G0))
2190  LET D2=D2-((P8↑G0*.161604*(1-X8↑2))/(3.93121*(1-P8↑G0)↑2))
2200  LET D2=D2*(1/(1-X8↑2)+(2*X8↑2)/((1-X8↑2)↑2))
2210  LET E=D1/D2
2220  LET A9=P8-E
2230  IF (ABS(P8-A9)<.000001) THEN 2255
2240  LET P8=A9
2250  GOTO 2150
2255  LET P8=A9
2260  RETURN
2270  PRINT# 1"INTERSTAGE LOSS LESS THAN ZERO"
2271  IF (Z0=0) THEN 1570
2272  GOTO 420
2280  LET C1=1
2290  LET K4=K4+.01
2300  LET Z0=0
2310  GOTO 291
2320  LET K4=1
2330  LET K6=K6+.01
```

```
2335  IF (K6>.82) THEN 2620
2336  LET Q8=0
2337  LET Q9=1
2340  LET K4=.4
2341  LET C1=1
2350  LET Z0=0
2360  GOTO 291
2370  IF (ABS(F2-F4)<.0005) THEN 1705
2375  GOTO 1706
2430  PRINT# 6"K1="K1"K4="K4"PT1/PT0="R9"K6="K6
2440  PRINT# 6"ROTOR FLOW COEFFICIENT="F2
2450  PRINT# 6"STATOR LOSS="Z0
2460  PRINT# 6"INTERSTAGE LOSS="Z3
2470  PRINT# 6"P1/PT0="P1
2480  PRINT# 6"P(I)/PT1="P5
2490  PRINT# 6"RPM="N
2602  GOTO 3000
2620  STOP
3000  LET Q0=R9/P7
3010  LET O=1-Q0↑G0
3020  LET S=(1-O)/O
3030  LET L0=(Q0/F2)↑2/O
3040  LET Z6=L0*(SQR(1+(2*S+1)/L0)-1)-S
3050  PRINT# 6"PREDICTED ROTOR LOSS="Z6
3060  LET Y2=SQR((1-.2514)*(1-Q0↑G0))
3070  LET X6=Y2*SQR(T4)
3080  LET U2=U3*(4.18375/4.195)
3090  LET T2=((1-.2514)*Q0↑G0+.2514)*T4
4000  LET X2=(U2-X6*SIN(71*J/180))↑2+X6↑2*(COS(71*J/180))↑2
4010  LET T1=T2+X2
4020  LET D3=T0*(1-T1)
4030  LET D4=T0*(1-R9↑G0)
4040  LET E0=D3/D4
4050  LET W0=F0*R0*V0*A0*K1
4060  LET H0=W0*.2402*(778/550)*D3
4070  LET M1=H0*550*(360/(J*N))
```

```
4080 LET D=P0/29.92
4090 LET O9=T0/518.7
5000 LET W1=W0*SQR(O9)/D
5010 LET H1=H0/(D*SQR(O9))
5020 LET M2=M1/D
5030 LET N0=N/SQR(O9)
5035 LET U1=(J*N*4.18375)/(360*V0)
5040 LET X0=(X1*COS(O1*J/180))↑2+(X1*SIN(O1*J/180)-U1*4.18375/4.195)↑2
5050 LET T5=1-X1↑2
5070 LET T6=1-X1↑2+X0↑2+(U1↑2*((4.2503/4.18375)↑2-1))
5080 PRINT# 6"CALCULATED EFFICIENCY="E0
5090 PRINT# 6"CALCULATED FLOW RATE="W0
5100 PRINT# 6"CALCULATED HORSEPOWER="H0
5110 PRINT# 6"CALCULATED ROTOR TORQUE="M1
5120 PRINT# 6"CALCULATED REFERRED FLOW RATE="W1
5130 PRINT# 6"CALCULATED REFERRED HORSEPOWER="H1
5140 PRINT# 6"CALCULATED REFERRED ROTOR TORQUE="M2
5150 PRINT# 6"CALCULATED REFERRED RPM="N0
5160 LET Z7=1-((T4-T2)/(T4-(R9/P4)↑G0))
5170 PRINT# 6"CALCULATED ROTOR LOSS COEFFICIENT="Z7
5171 PRINT# 6" "
5172 PRINT# 6" "
5180 GOTO 1570
7000 LET S8=.001
7010 LET S7=SQR((1-S9)*(1-S8↑G0))
7020 LET A9=S8*(S7/(1-S7↑2))-↑2
7030 LET D1=(S8↑G0)*G0/2*((1-S7↑2)/(1-S8↑G0))
7040 LET D1=(S7/(1-S7↑2))*(1-(D1*(1/(1-S7↑2)+(2*S7↑2)/(1-S7↑2)↑2)))
7050 LET E=A9/D1
7060 LET A8=S8-E
7070 IF (ABS(S8-A8)<.0001) THEN 7100
7080 LET S8=A8
7090 GOTO 7010
7100 LET S8=A8
7110 LET S7=SQR((1-S9)*(1-S8↑G0))
7120 RETURN
```

TABLE A-I

DEFINITION OF VARIABLES

A0	(A^*) Area at the Stator Throat
A1	(A_1) Area at the Stator Exit Plane
A8	Decision Variable
A9	Decision Variable
C1	Decision Variable
D	$(\delta) = P_{to}/P_{ref}$
D1	$\frac{d\phi}{d\left(\frac{P}{P_t}\right)}$
D2	$\frac{d^2\phi}{d\left(\frac{P}{P_t}\right)^2}$
D3	(ΔT_w)
D4	(ΔT_{is})
E	Decision Variable
E0	(η_{T-S}) Total-to-Static Efficiency
F0	(ϕ^*) Flow Coefficient at Stator Throat
F1	(ϕ_1) Flow Coefficient at Stator Exit Plane
F2	(ϕ_R) Flow Coefficient Through Rotor Passage
F3	(ϕ_i) Flow Coefficient Through Interblade Area
F4	Input Value of ϕ_R Based on Assumed Rotor Loss
F8	Decision Variable
F9	Decision Variable
G0	(Gamma - 1)/Gamma
H0	Horsepower
H1	Referred Horsepower
H8	(ρ) Rho
H9	Reynolds Number

J	(π)	Pi
K1	k^*_{bs}	Blockage Factor at Stator Throat
K4	k_{bi}	Interblade Blockage Factor
K5	k_{bR}	Blockage Factor in the Rotor
K6	k_{b1}	Blockage Factor at the Stator Exit Plane
M1		Rotor Torque
M2		Referred Rotor Torque
N		RPM
NO		Referred RPM
O1	(α_1)	Flow Angle at Stator Exit Plane
O3	(α_i)	Flow Angle at Rotor Entrance Plane
O4	(Δk_{bi})	Increment for k_{bi}
P	P_{ti}/P_{to}	
PO	P_{to}	
P1	P_1/P_{to}	
P2	P_2/P_{to}	
P3	P_1/P_{t1}	
P4	P_{t1}/P_{to}	
P5	P_i/P_{t1}	
P6	P_i/P_{to}	
P7	P_{Ei}/P_{to}	
P8		Decision Variable
P9	P_2	
R	(ρ_{to})	Total Density at Stagnation Conditions
R7		Relaxation Parameter
R8		Relaxation Parameter
R9		Predicted P_2/P_{to}

S7-S9	Used in GOSUB 7000 to Find P_2/P_{Ei}
T0	T_{to}
T1	T_{t2}/T_{to}
T2	T_2/T_{to}
T3	T_i/T_{to}
T4	T_{Ei}/T_{to}
T5	T_1/T_{to}
T6	T_{E1}/T_{to}
U1	$\overline{U}_1$ Dimensionless Velocity (See Appendix A)
U2	$\overline{U}_2$ Dimensionless Velocity (See Appendix A)
U3	$\overline{U}_i$ Dimensionless Velocity (See Appendix A)
V0	V_{to} Total Velocity
W0	$\dot{w}$ Flow Rate
W1	$\dot{w}^*$ Referred Flow Rate
W9	Decision Variable
X0	X_{w1} Non-Dimensional Relative Velocity
X1	X_1 Non-Dimensional Velocity
X2	X_2 Non-Dimensional Velocity
X3	X_i Non-Dimensional Velocity
X4	X_{ui} Non-Dimensional Velocity
X5	X_{wi} Non-Dimensional Relative Velocity
X6	X_{w2} Non-Dimensional Relative Velocity
X8	Decision Variable
Y2	Non-Dimensional Velocity
Y6	Increment for k_{b1}
Y7	Increment for k_{bi}
Y8-Y9	Counters

Z0	z_s Stator Loss Coefficient
Z3	z_i Interblade Loss Coefficient
Z7	z_{1-2} Predicted Rotor Loss Coefficient
Z8	Increment for Stator Loss Coefficient
Z9	Increment for Interblade Loss Coefficient

FIGURE A-1 PRESSURE DISTRIBUTION THROUGH THE STATOR

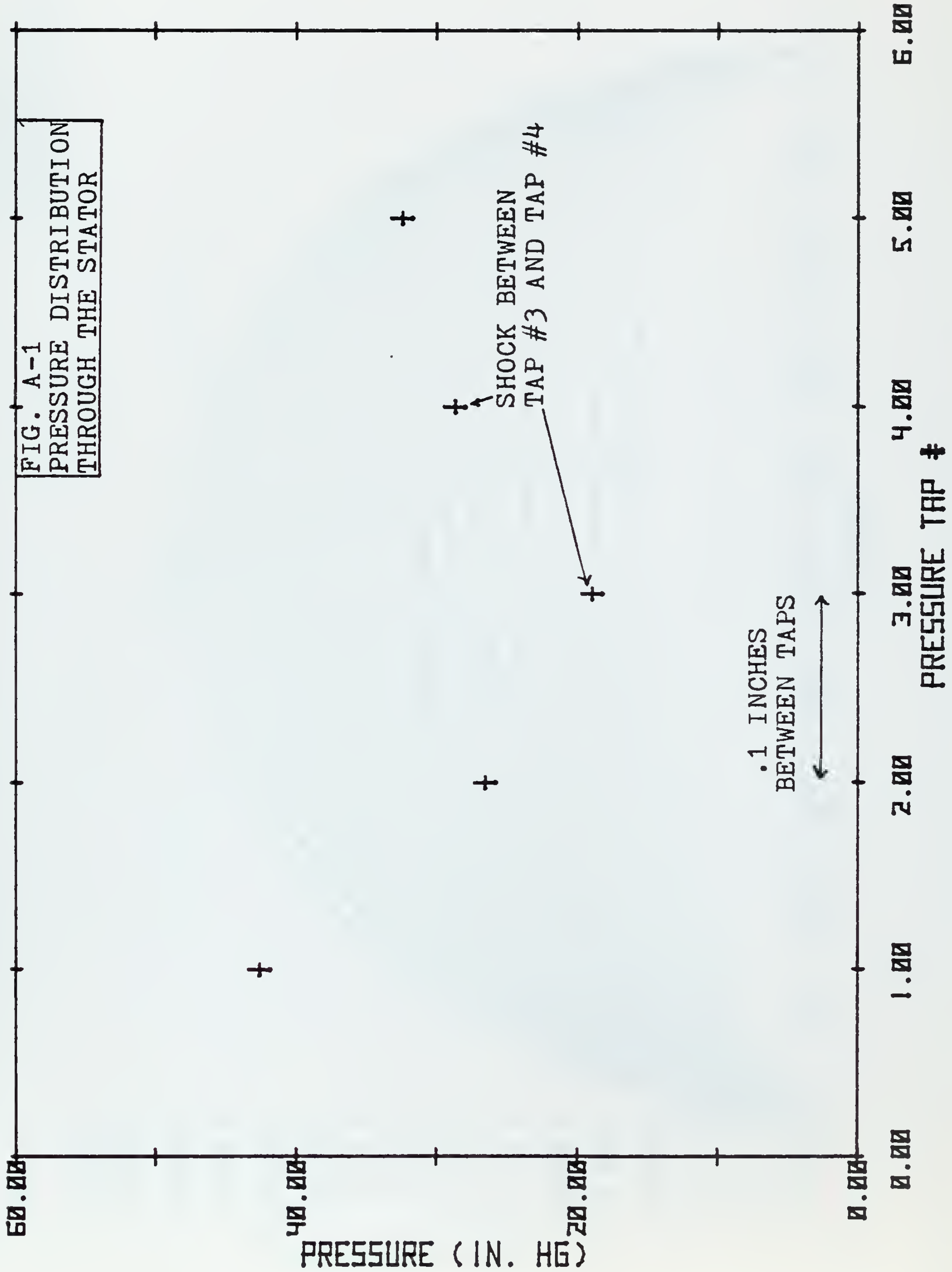


FIGURE A-2 ILLUSTRATION OF SOLUTION AREA

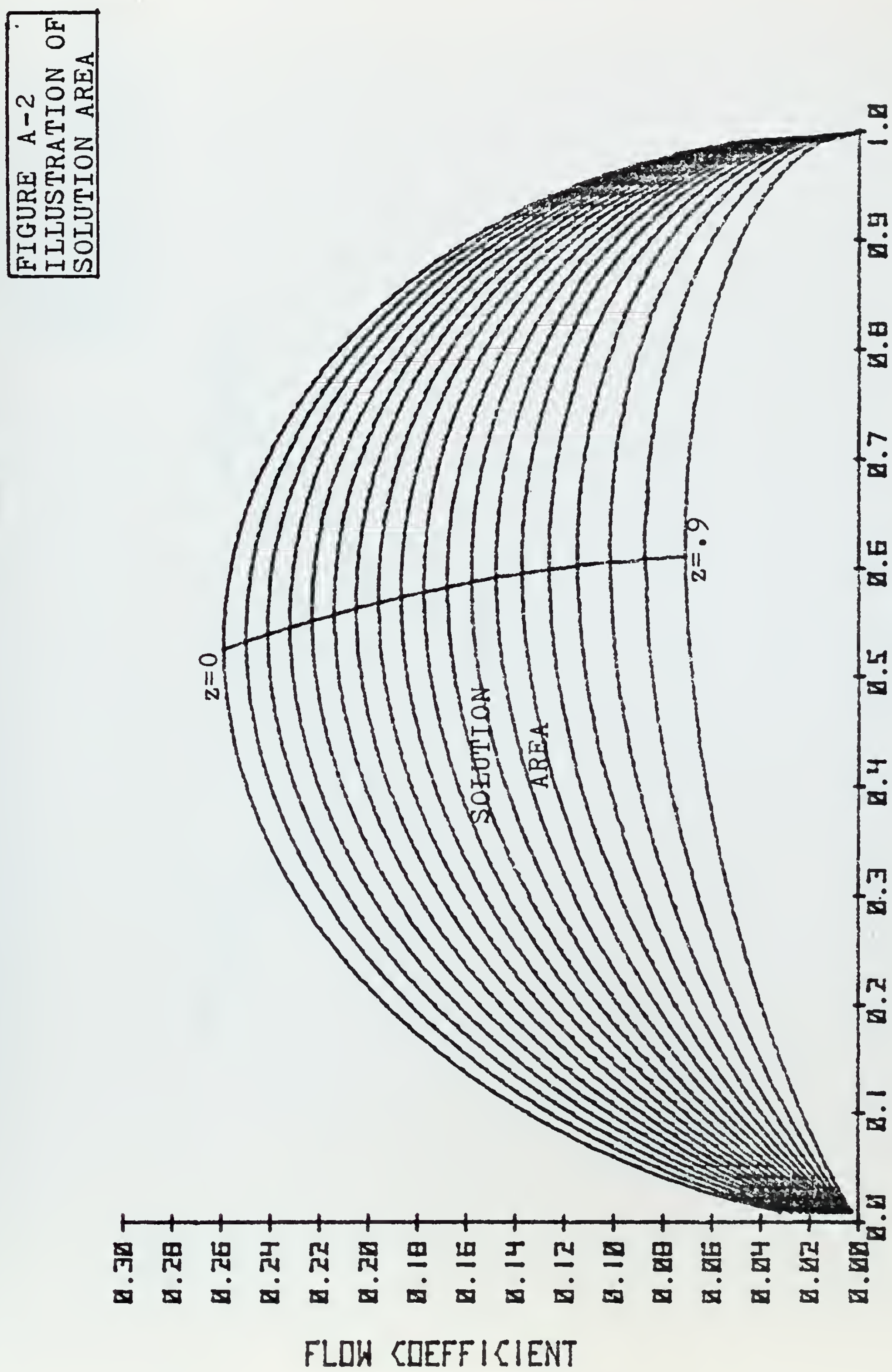


FIGURE A-3 MAXIMUM Ø AND SONIC LINE

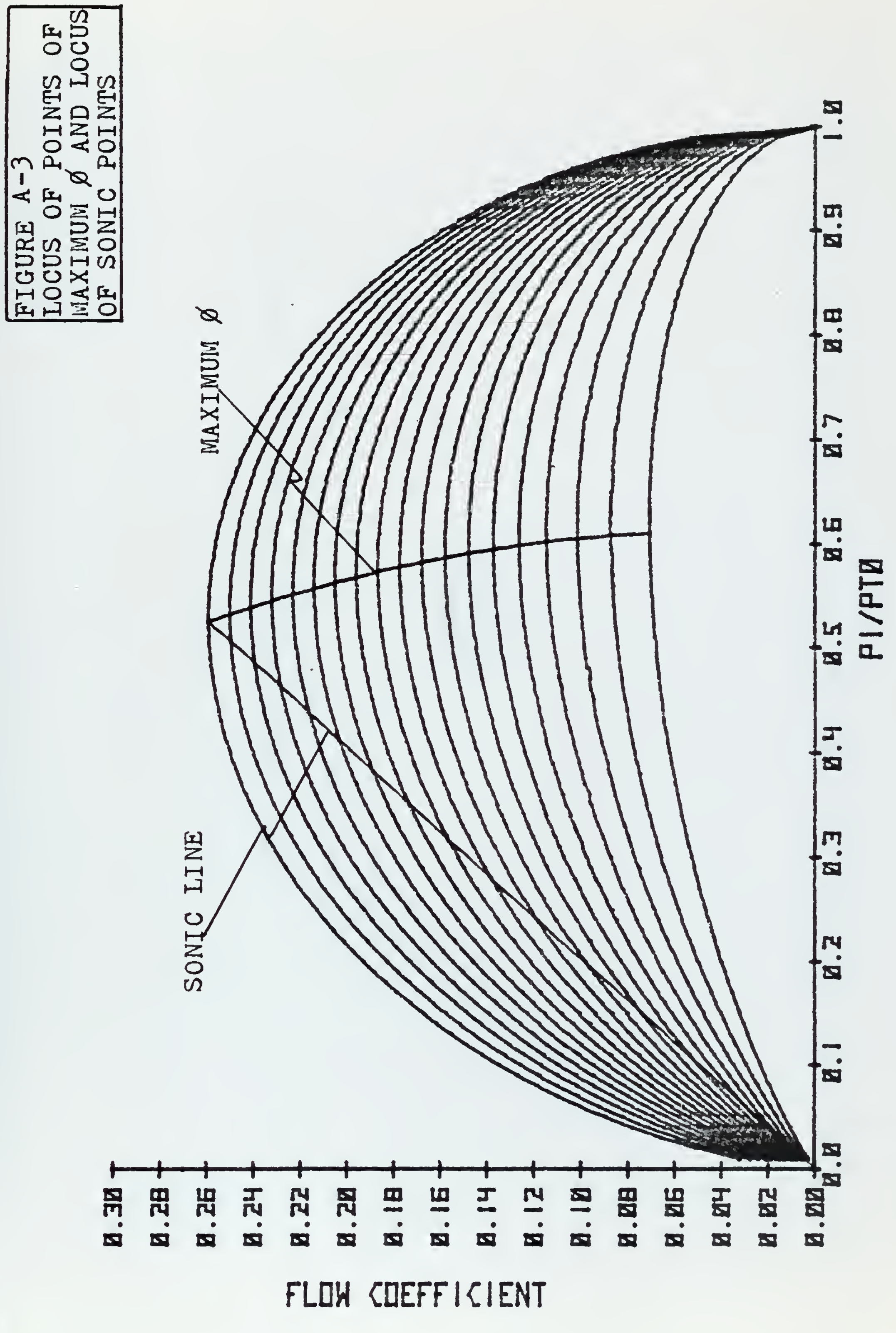
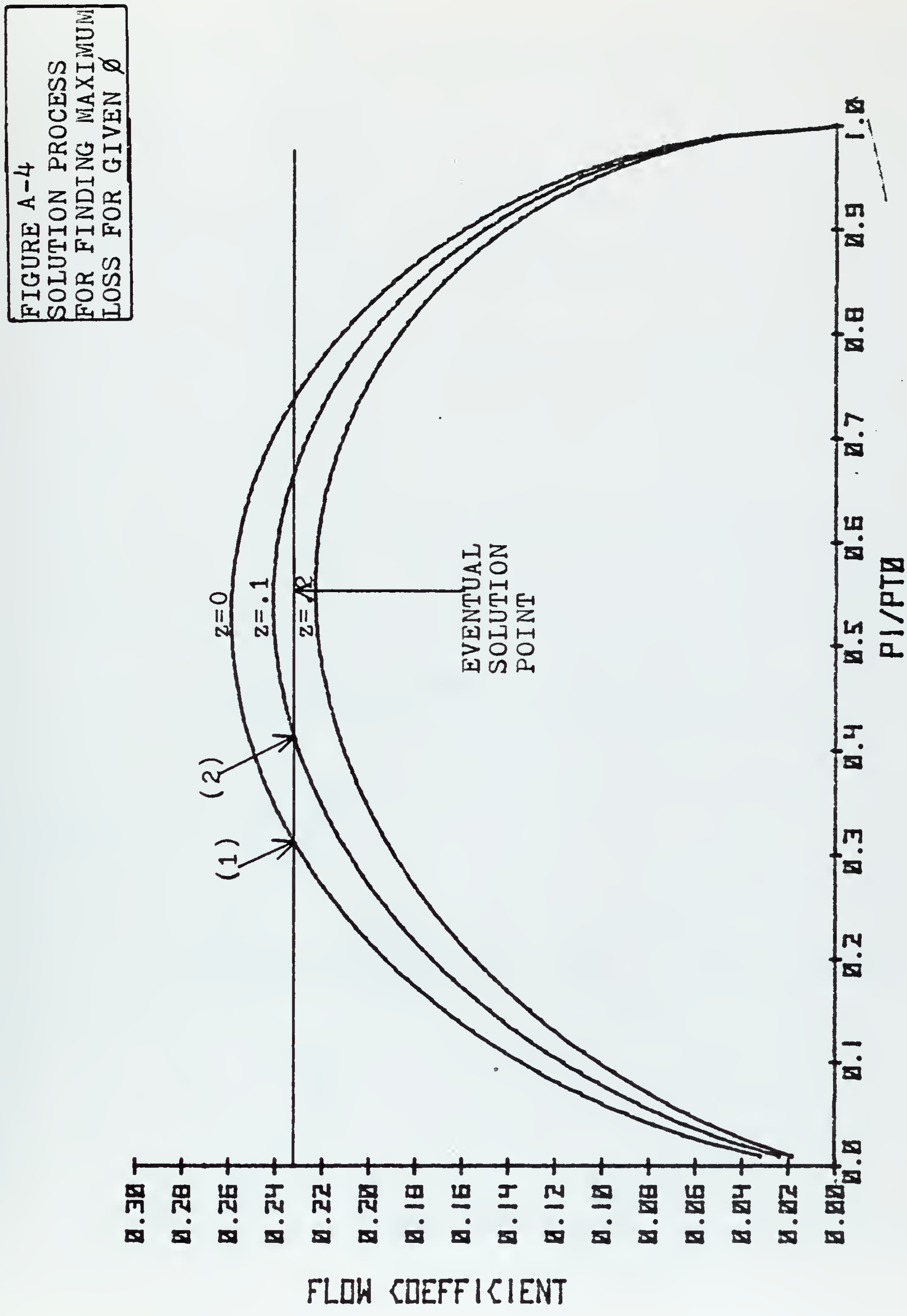


FIGURE A-4 TYPICAL SOLUTION PROCESS



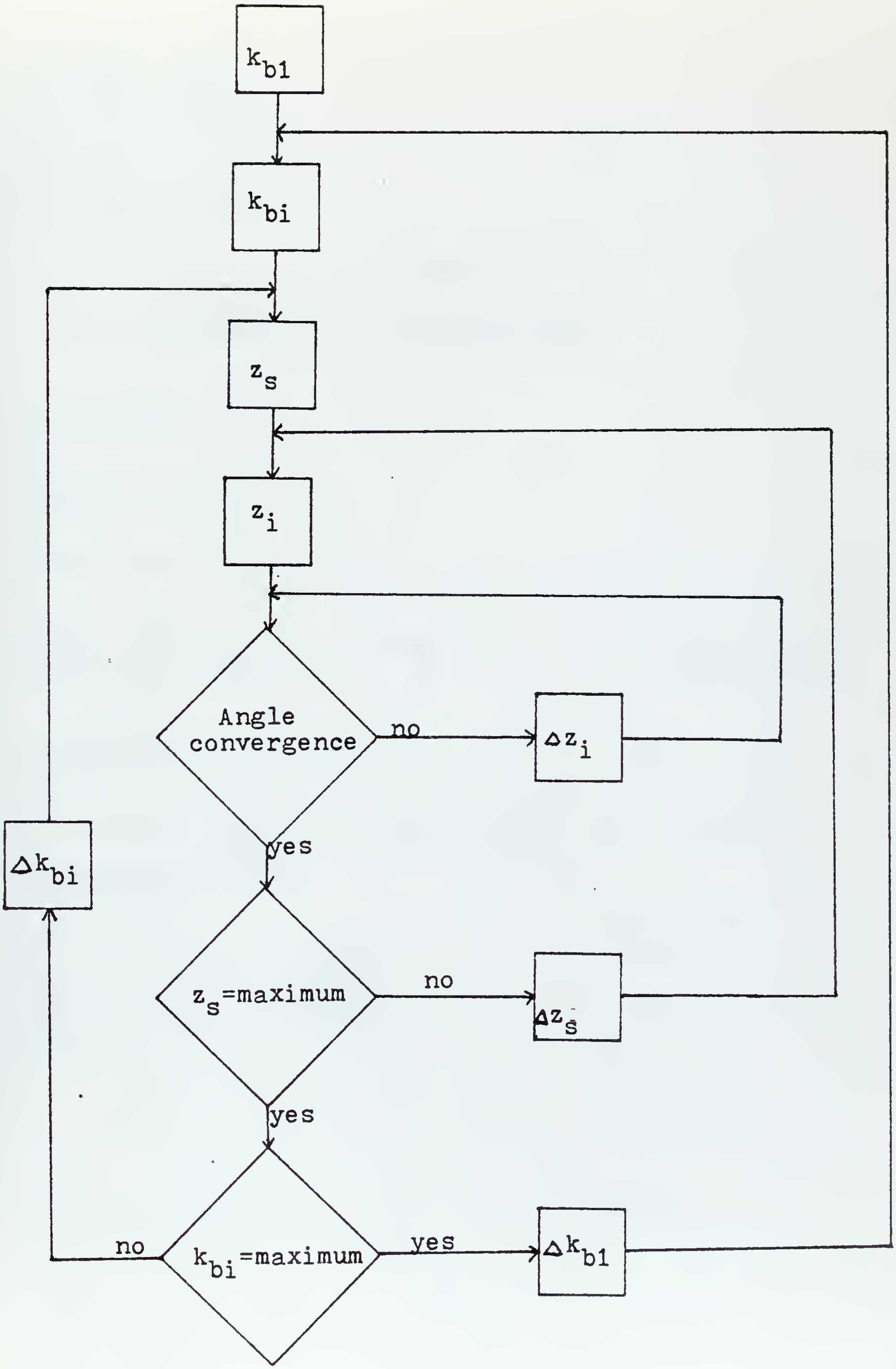


FIG. A-5 PROGRAM SCHEMATIC

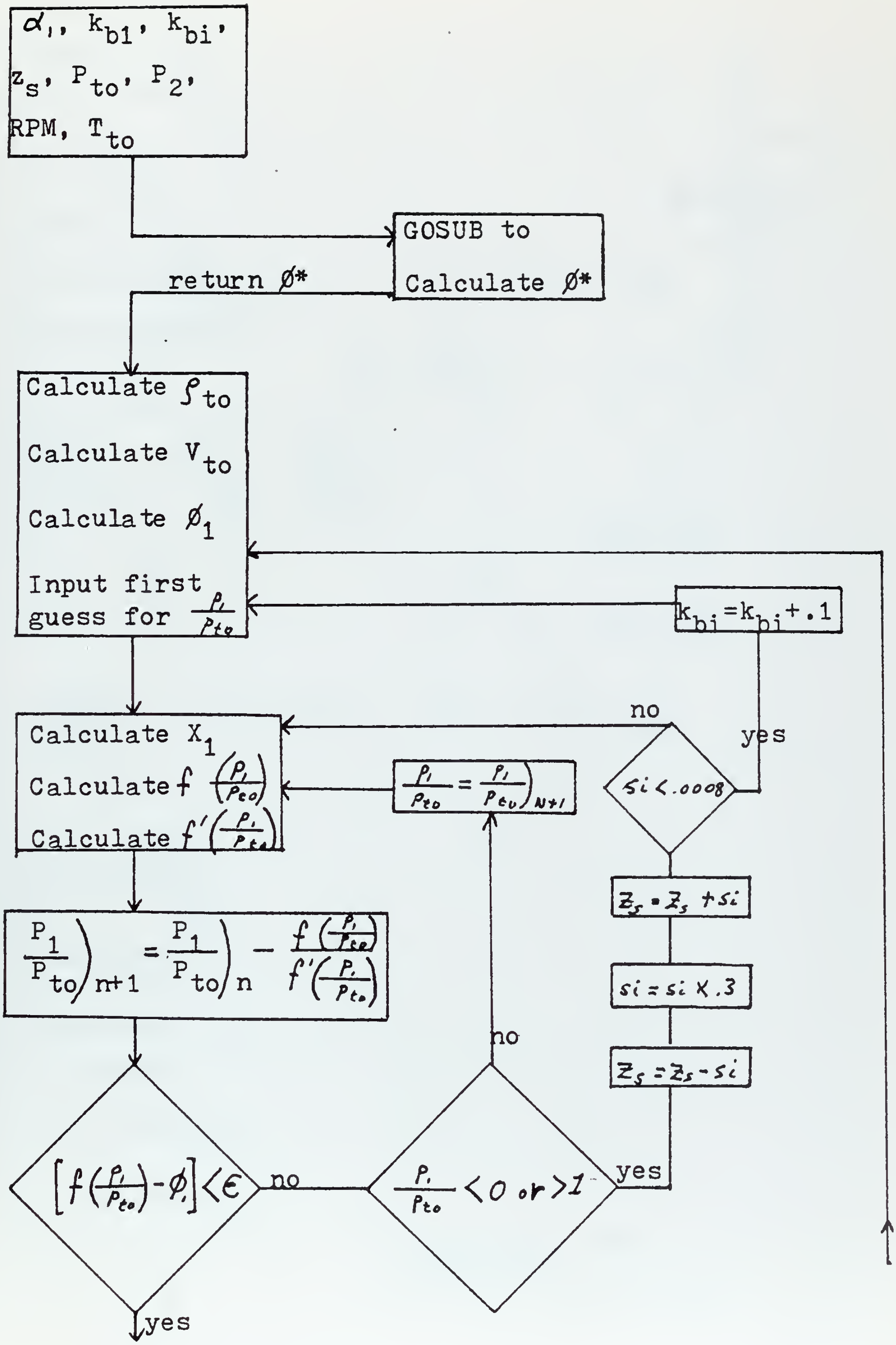
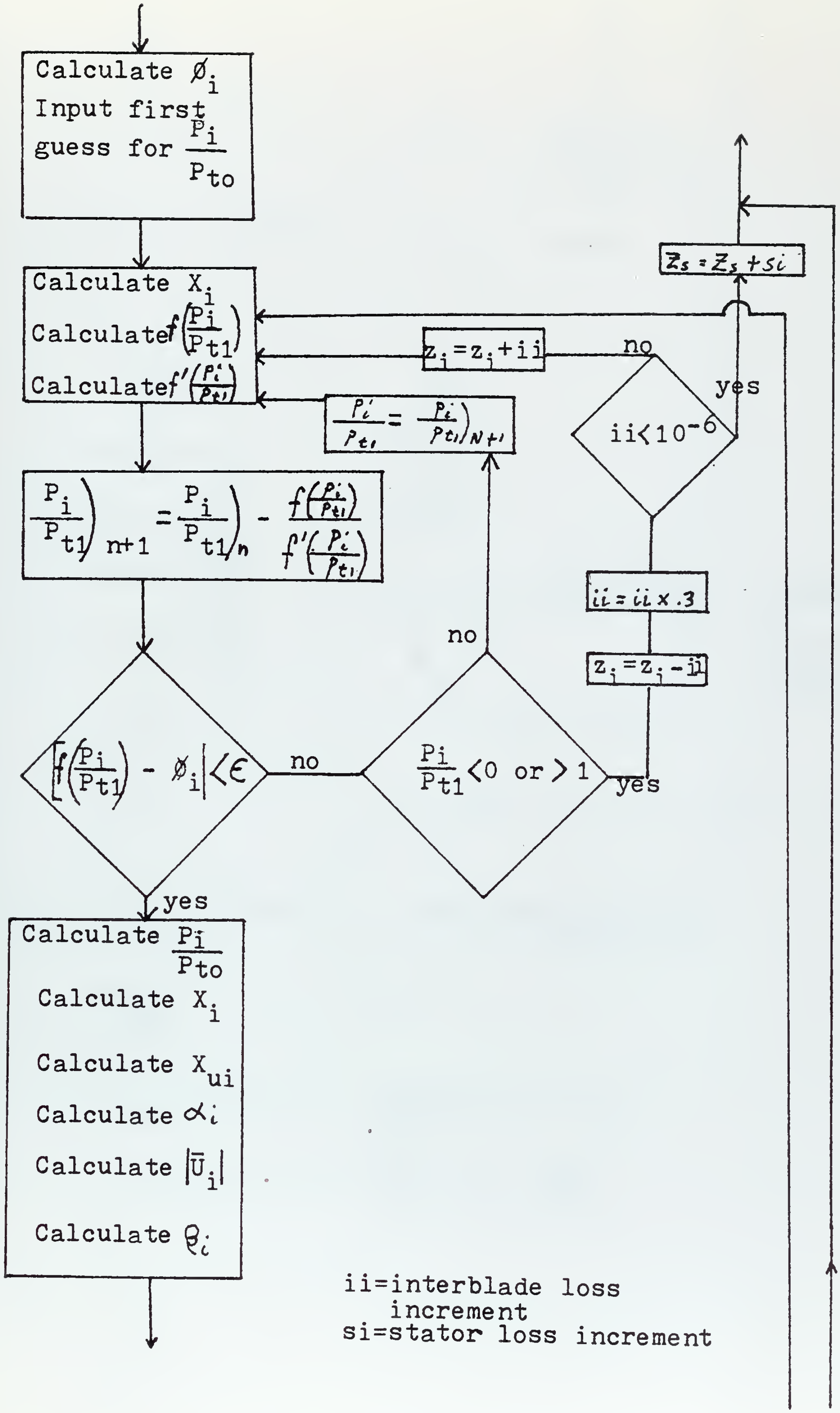
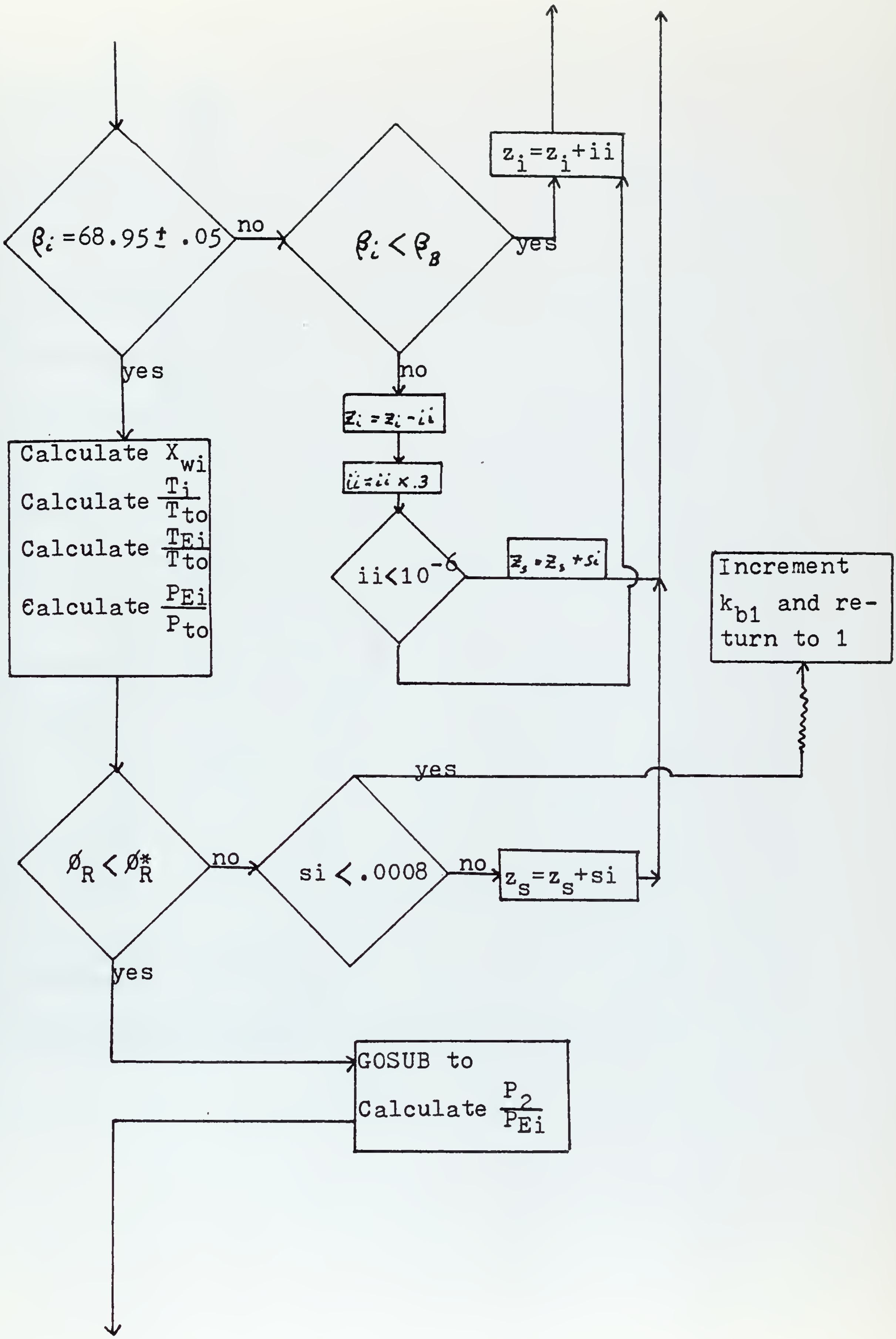


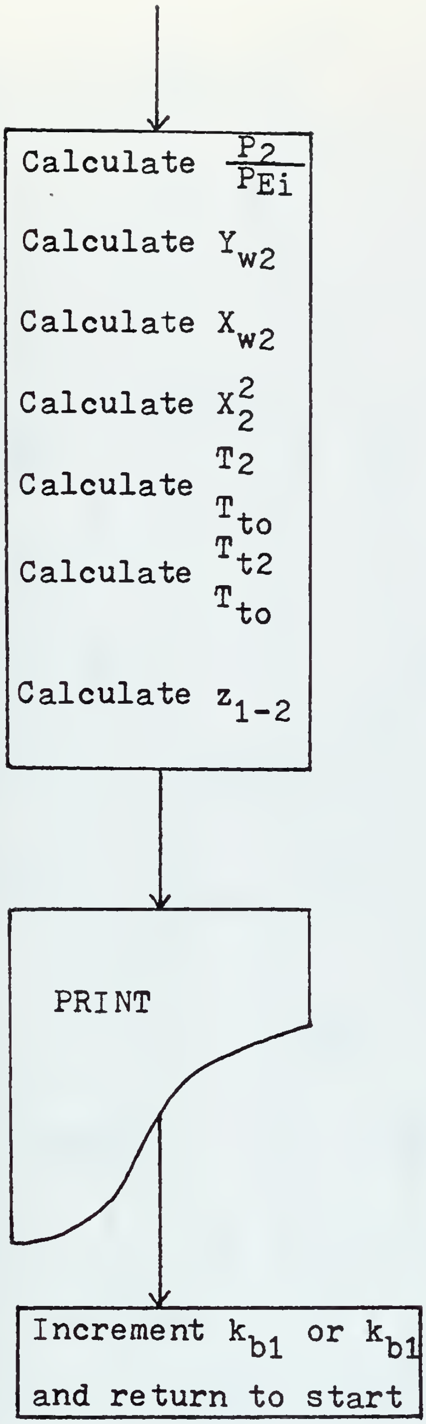
FIG. A-6 FLOW CHART



ii=interblade loss increment
si=stator loss increment



$\beta_i = 68.95 \pm .05$
no
yes
$\beta_i < \beta_B$
yes
no
$z_i = z_i + ii$
$z_i = z_i - ii$
$ii = ii \times .3$
$ii < 10^{-6}$
$z_s = z_s + si$
Calculate X_{wi}
Calculate $\frac{T_i}{T_{to}}$
Calculate $\frac{T_{Ei}}{T_{to}}$
Calculate $\frac{P_{Ei}}{P_{to}}$
Increment k_{b1} and re- turn to 1
$\phi_R < \phi_R^*$
no
$si < .0008$
yes
no
$z_s = z_s + si$
yes
GOSUB to Calculate $\frac{P_2}{P_{Ei}}$


Calculate $\frac{P_2}{P_{Ei}}$
Calculate Y_{w2}
Calculate X_{w2}
Calculate X_2^2
Calculate $\frac{T_2}{T_{to}}$
Calculate $\frac{T_{t2}}{T_{to}}$
Calculate z_{1-2}
PRINT
Increment k_{b1} or k_{b1}
and return to start

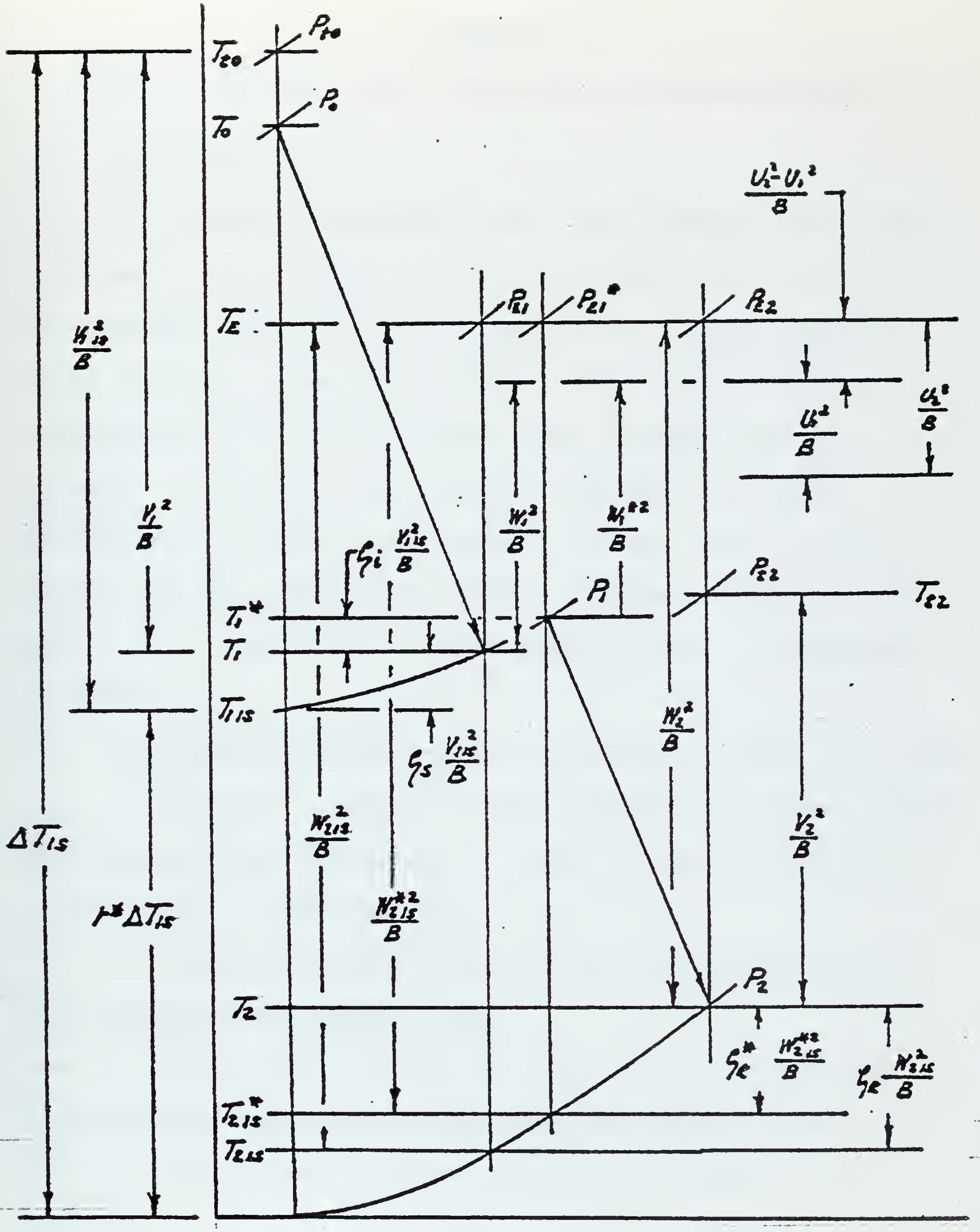


FIGURE A-7 THERMODYNAMIC PROCESS OF FLUID IN AN AXIAL TURBINE STAGE

APPENDIX B

TURBINE TEST RIG (TTR) DATA REDUCTION AND PROCESSING

B-1 INTRODUCTION

This appendix describes only those changes that have been made to the data reduction procedure given in Ref. 2. The program numbers and the functions of those programs given in Ref. 2 have not changed. The channel and port assignments for data collection and storage have not changed. Variables also have not changed, although additional variables were defined in the course of modifying the data reduction process. Those variables that now exist in addition to those given in Ref. 2 are given in Table B-I.

Modifications made during the course of this work were:

1. Raw data can now be read directly from mass memory. Previously it was necessary to read the paper tape for a given point in order to reduce it.

2. Provisions have been written into the programs for smoothing the reduced data.

3. All of the parameters used in calculating the theoretical loss coefficients that were previously entered from charts are now in polynomial form in the program.

4. Storage of raw data is now a separate program. Previously it was necessary to run the reduction sequence to store data.

B-2 DESCRIPTION OF PROGRAMS

The data reduction is divided into ten separate programs with an additional program to store the raw data. Fig. B-1 shows the contents of each program and also shows the sequence in which the programs are chained. Following is a description of the reduction sequence referred to Fig. B-1 which will be followed by a description of how to run the reduction program.

TTR in Fig. B-1 is used only to store the raw data. TTR does not chain to any other program.

The data reduction process begins in TTR1. In order to smooth the reduced data as discussed in Section III it is necessary to have n values for $(P_1-P_{hub})/(P_{tip}-P_{hub})$. Therefore, the first step in the data reduction process is to go from TTR1 to the point in TTR2 where the values of P_1, P_{tip}, and P_{hub}, have been calculated. On completion, the program returns to TTR1 and repeats the process n times until the n values of $(P_1-P_{hub})/(P_{tip}-P_{hub})$ have been calculated. The program then proceeds to TTR9 where a polynomial curve fit for the n values of $(P_1-P_{hub})/(P_{tip}-P_{hub})$ is generated. Then the program returns the polynomial coefficients to TTR1B where the reduction process begins at the first point in the run. However, when the point in TTR2 is reached where P_1/P_{to} is calculated, it is calculated using the polynomial as described in Section III.

The rest of the reduction process in Fig. B-1 is the

same as described in Ref. 2 with the exception that no further keyboard inputs are needed until the raw and reduced data is tabulated.

B-3 RAW DATA STORAGE

The procedure for storing raw data on the mass memory is outlined in Ref. 2, Appendix B. Sec. B-3, lines 10-19. Note that line 10 should read, "GET 'TTR'". Although it is now a separate program the procedures for storing data have not changed.

B-4 PROCEDURE FOR DATA REDUCTION PROGRAM

1. Key in GET "TTR1B"
2. Press "RUN EXECUTE".
3. The following check list will be printed on the HP 9830 printer:

"PRIOR TO RUNNING THIS SEQUENCE ENSURE FOLLOWING:

"THAT TTR2 LINE 590 HAS PROPER FACTOR IN IT;

1.02 FOR HOODED

1.01 FOR UNHOODED

"IF IT IS NECESSARY TO INPUT ALPHA 1 THEN CHANGE TTR2

"LINE 960 TO 'INPUT A3'

"IF BLOCKAGE FACTOR OTHER THAN 1, CHANGE TTR2-1060

"TO 'INPUT X7', AND PUT SEMICOLON AFTER TTR2-1050

"ENSURE THAT TTR1, TTR1B HAVE THE PROPER FILENAME IN 441

"IF IT ISN'T DESIRED TO STORE REDUCED DATA CHANGE

TTR6-580 TO 'G1=0'

"ENSURE TTR6-610, 630 HAVE PROPER FILENAME FOR REDUCED

DATA

"ENSURE TTR7-220 HAS PROPER FILENAME FOR RAW DATA

"ENSURE TTR8-220 HAS PROPER FILENAME FOR REDUCED DATA

"IF IT IS DESIRED TO RUN A SINGLE POINT, WITHOUT SMOOTHING, INSERT TTR2-1127 'GOTO 1200'. OTHERWISE OMIT TTR2-1127

The above checklist will prepare all ten programs. The last item is a provision for elimination of smoothing if it is desired to run a single point. If smoothing is not desired then each point must be run separately with TTR2-1127 inserted.

4. Press CONTINUE EXECUTE

5. The display will read LOWEST, HIGHEST RECORD # THIS RUN. Input the lowest record number and the highest record number from which it is desired to read raw data. Note, the record number on which the reduced data will be stored will be the same number as the raw data record from which the raw data was read. Make sure the filename designated for the storage of the reduced data can accept data on those record numbers without writing over previous information.

6. Next the display will read PRINT OUT RESULTS? YES=1, NO=0. If a 1 is selected here then at the conclusion of data reduction, for all the points in the run, the program will begin to print all results. If 0 is selected then all reduced data will be stored on the mass memory.

7. It will take about 15 minutes for the program to read the raw data and make the calculations necessary for

smoothing the data. When this is oompleted the program will start reducing each point and printing the results.

8. At the completion of data reduction for all points the option to tabulate the data is available. ENTER RECORD #'S: LOWEST, HIGHEST will appear on the display. If it is desired to tabulate the raw data then enter the same record numbers that were entered to initiate the program. The raw data will then be tabulated.

9. Next the display will read ENTER LOWEST, HIGHEST RECORD NUMBER. If it is desired to tabulate the reduced data then enter the appropriate record numbers. The reduced data will be tabulated.

TABLE B-I

VARIABLES ADDED TO DATA REDUCTION PROGRAM

A2	Lowest Record Number
A5	Counter
A6	Decision Variable
A7	Highest Record Number
B	(Array) Polynomial Coefficients From TTR9
F8	Decision Variable
Q	(Array) Values of Isentropic Head Coefficient
T4	Polynomial Approximation of Reynolds Number
Z	Decision Variable

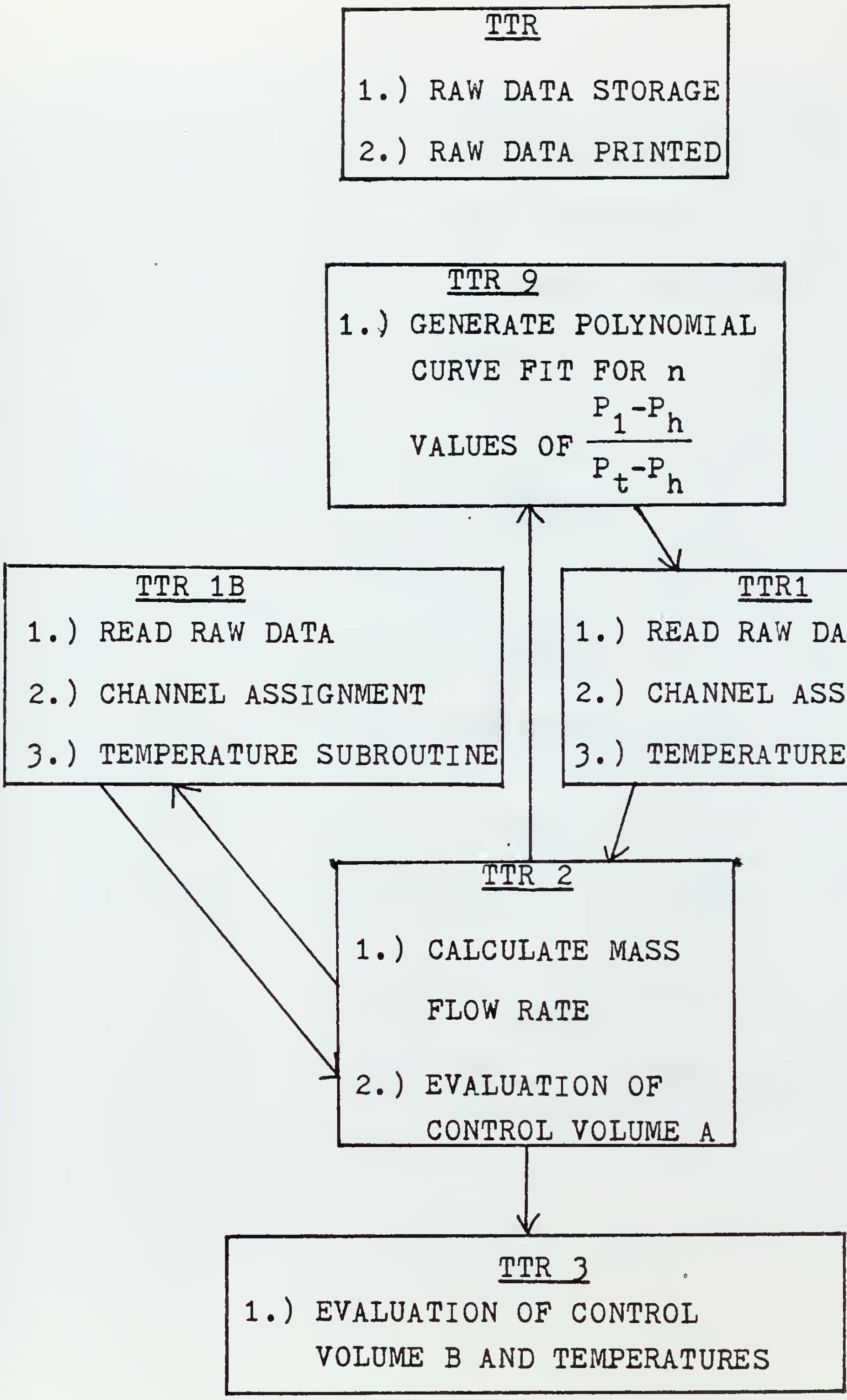
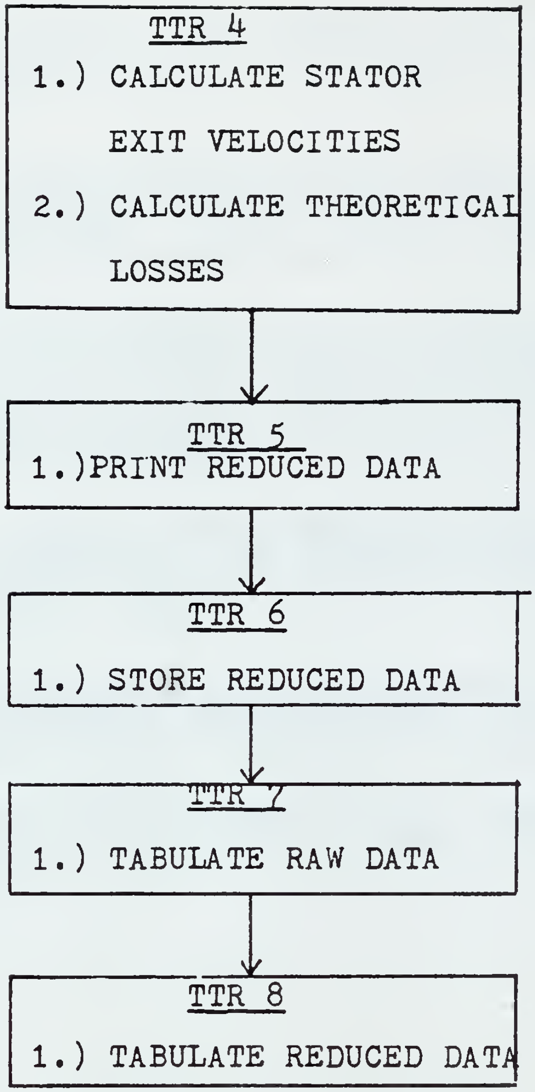


FIGURE B-1 DATA REDUCTION SCHEMATIC


TTR 4
1.) CALCULATE STATOR EXIT VELOCITIES
2.) CALCULATE THEORETICAL LOSSES
TTR 5
1.) PRINT REDUCED DATA
TTR 6
1.) STORE REDUCED DATA
TTR 7
1.) TABULATE RAW DATA
TTR 8
1.) TABULATE REDUCED DATA

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c.1 Analytical and experimental determination of the characteristics of a transonic axial turbine.